진공실무수련회

컨덕턴스 계산
(고진공, 원형도관을 중심으로)

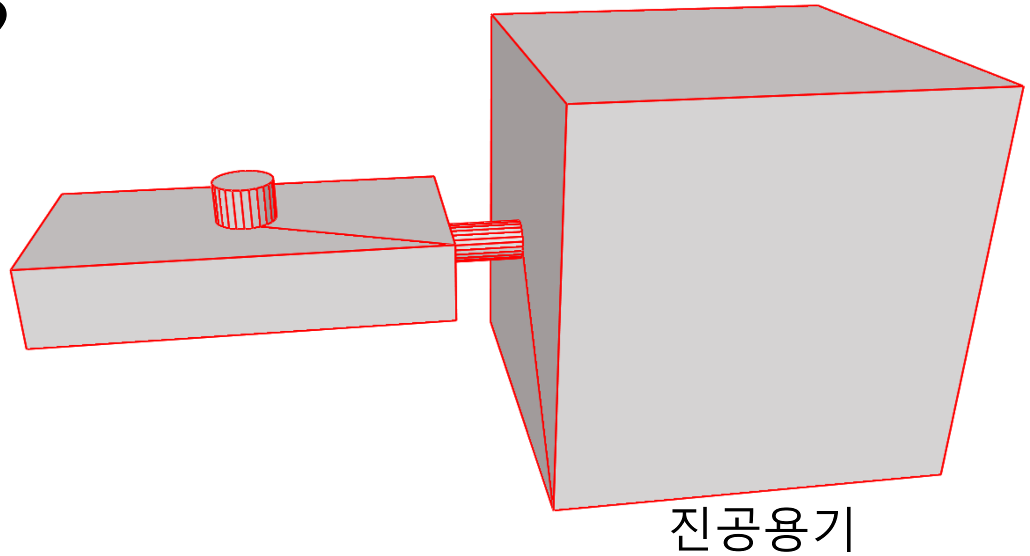
2020. 11. 5

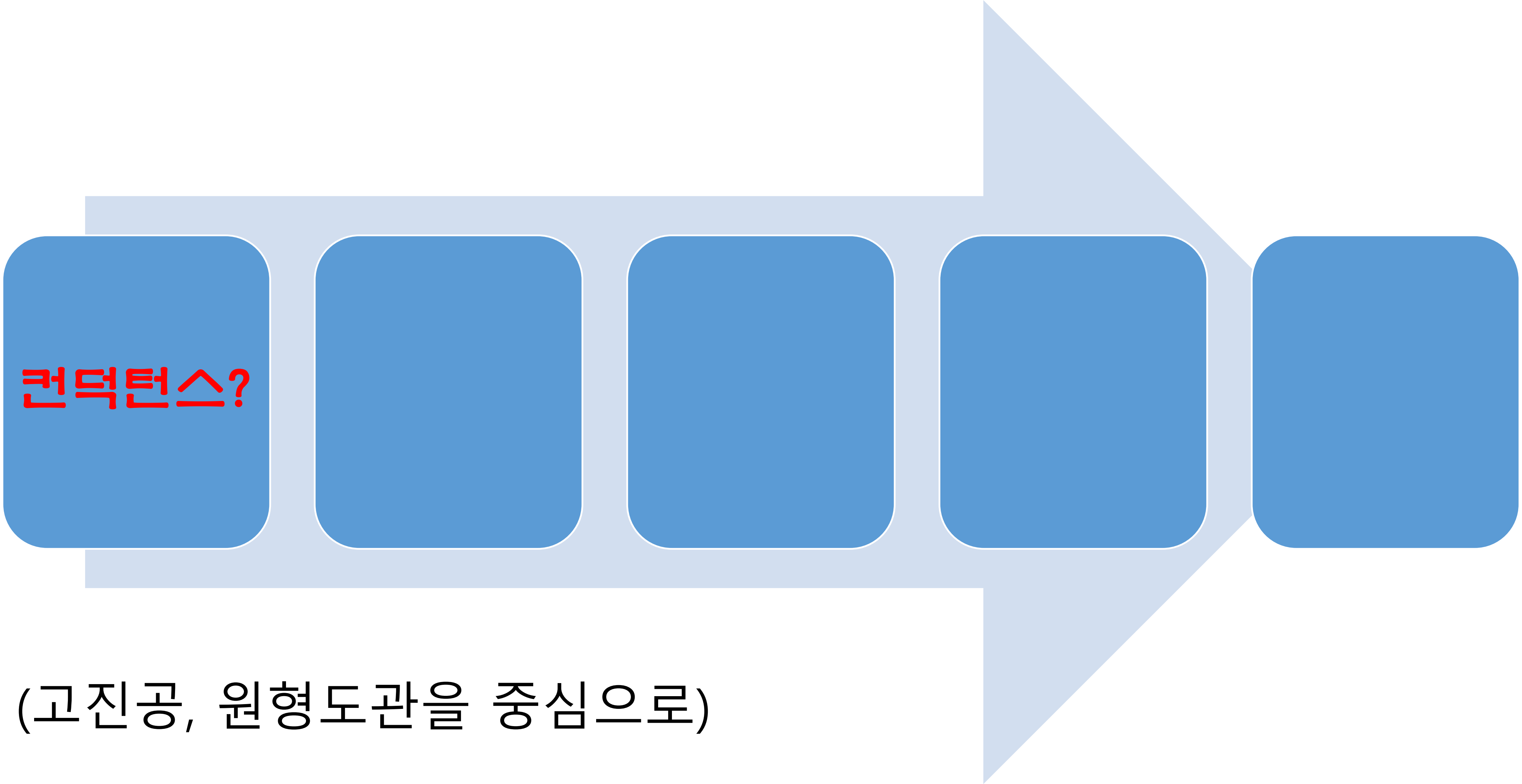
13:00 – 14:30

박 종도

포항가속기연구소

Conductance 계산은 왜 하나요?





(고진공, 원형도관을 중심으로)

- 벽을 두드리는 입자수

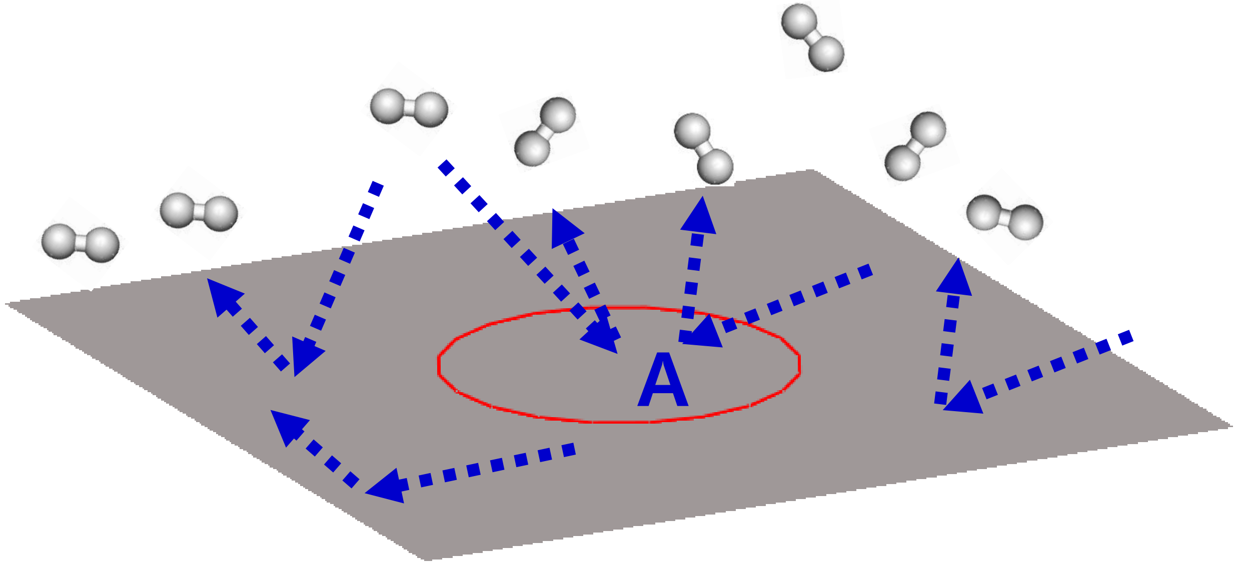
$$\Phi = \frac{1}{4} n v_a \quad [\mathrm{m^{-2}s^{-1}}]$$

- 벽을 두드리는 입자의 부피

$$\frac{\Phi}{n} = \frac{1}{4} v_a \quad [\mathrm{m\,s^{-1}}]$$

$$= 116 \quad \mathrm{m\,s^{-1}}$$

$$= 11.6 \quad \mathrm{L\,s^{-1}cm^{-2}}$$



23°C, air
A=1 cm²

- 단위면적을 빠져나가는 기체의 부피

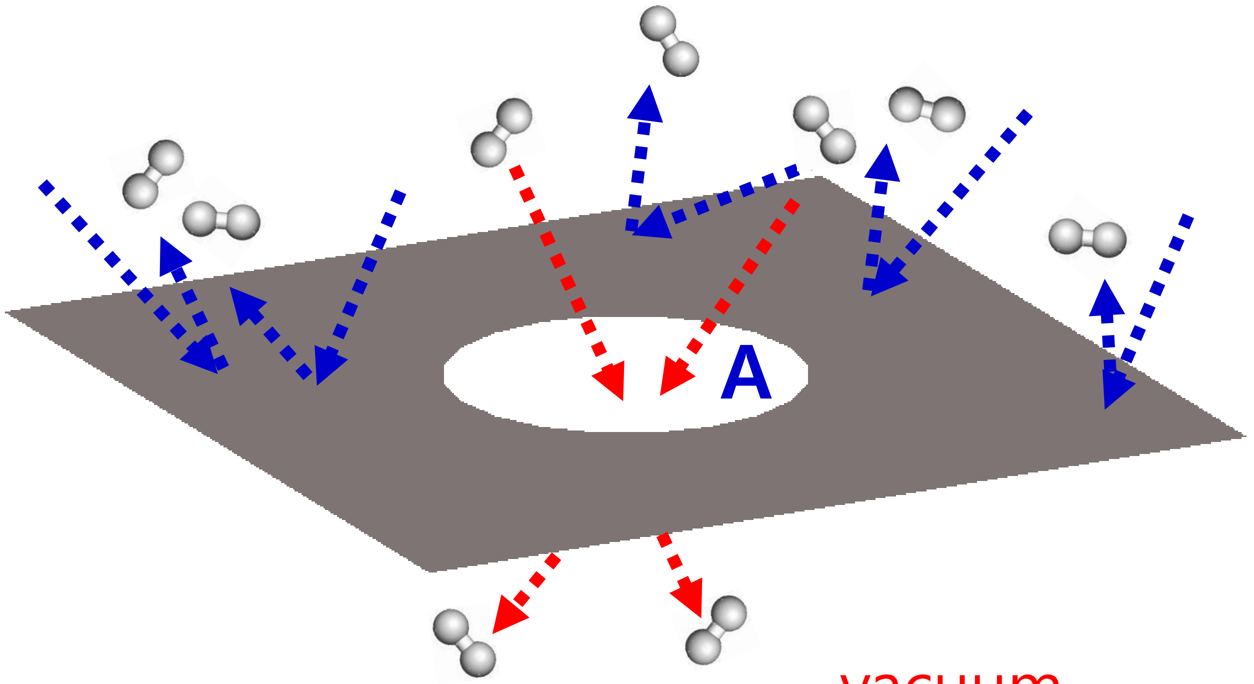
$$\frac{\Phi}{n} A = \frac{1}{4} v_a A$$

$$= 11.6 \ \ \mathrm{L\ s^{-1}}$$

(23°C, air, A=1 cm²)

- $C_o = 11.6\ A \quad \mathrm{L\ s^{-1}}$

(A cm²)



도관을 연결 할 때

- 단면적을 빠져나가는 부피

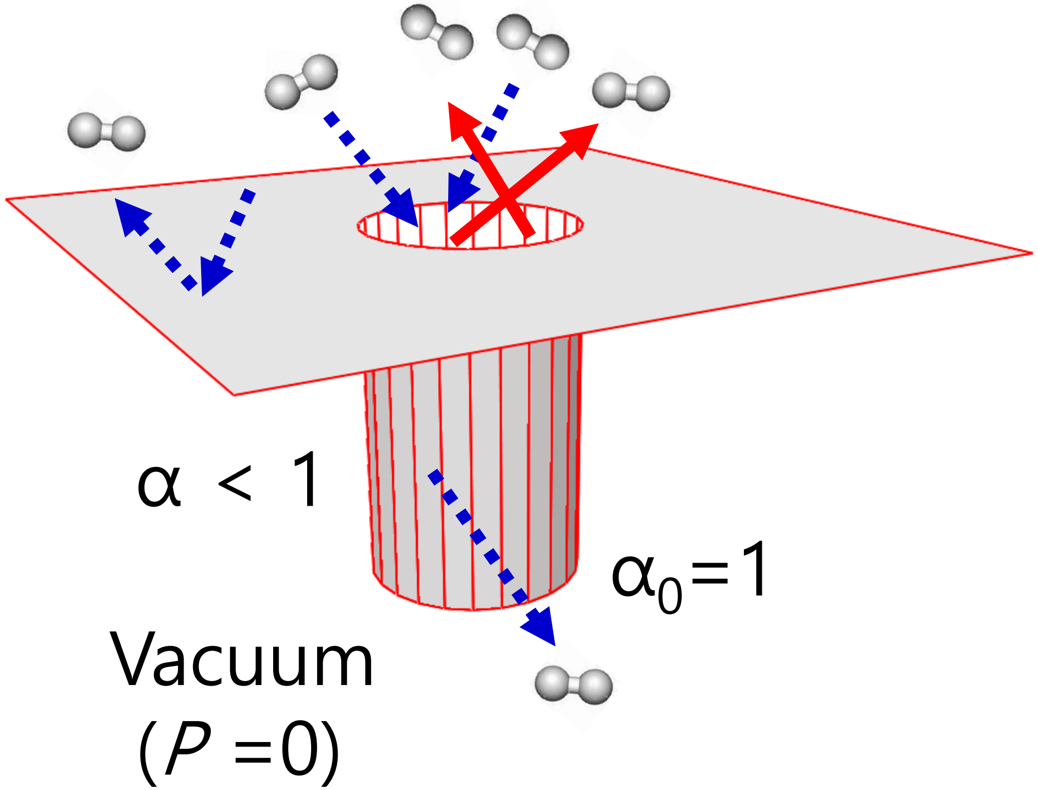
$$\frac{\Phi}{n} A = \alpha \cdot \frac{1}{4} v_a A$$

$$= \alpha \cdot 11.6 \quad L\,s^{-1}$$

($23^{o}C$, air, A=1 cm^2)

- C=11.6 **α**·A $L\,s^{-1}$

(**α** 통과 확률)

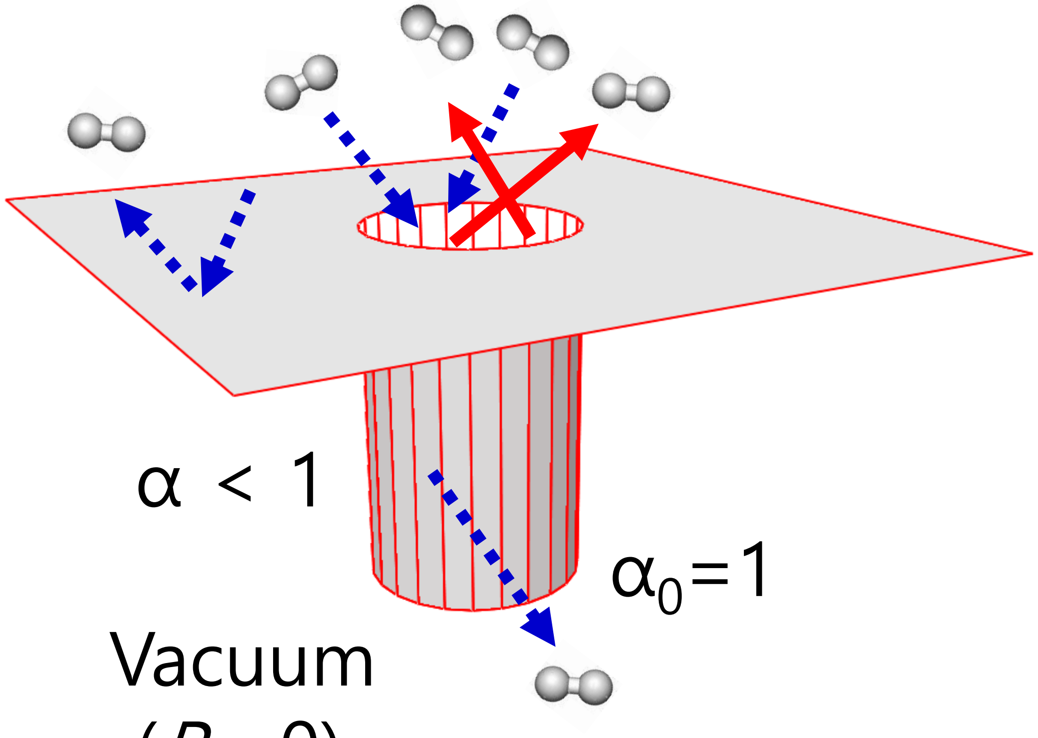


- 단면적을 빠져나가는 부피, 즉 컨덕턴스는
  - 단면적이 클수록
  - 길이가 짧을 수록 커진다

(주의)

- 단면적을 빠져나가는 기체의 양, 즉 유량은 그 부피에 들어있는 기체의 압력에 비례

$$Q = C\ \Delta P \qquad (\mathrm{mbar\ L\ s^{-1}})$$

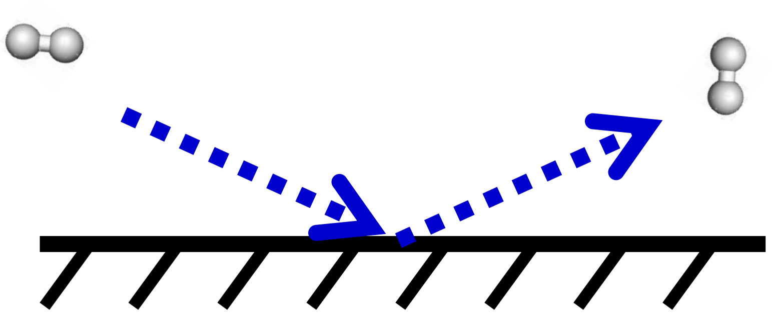


$23^{o}C$ 공기는 (단위 cm) $C = 11.6\ \alpha A$ L/s

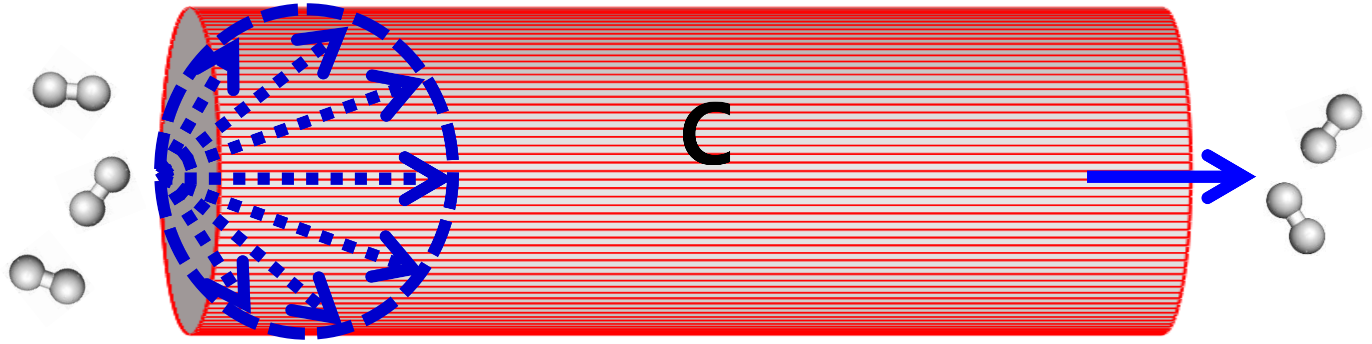
다른 기체는 $C = \beta\ 11.6\ \alpha A$ L/s

$$= \sqrt{\frac{29}{m}}\ 11.6\ \alpha A \quad L/s$$

	질량(M)	공기에 대한 비율 β
수소(H_2)	2	3.81
헬륨(He)	4	2.69
질소(N_2)	28	1.02
산소(O_2)	32	0.952
알곤(Ar)	40	0.851



거울 반사

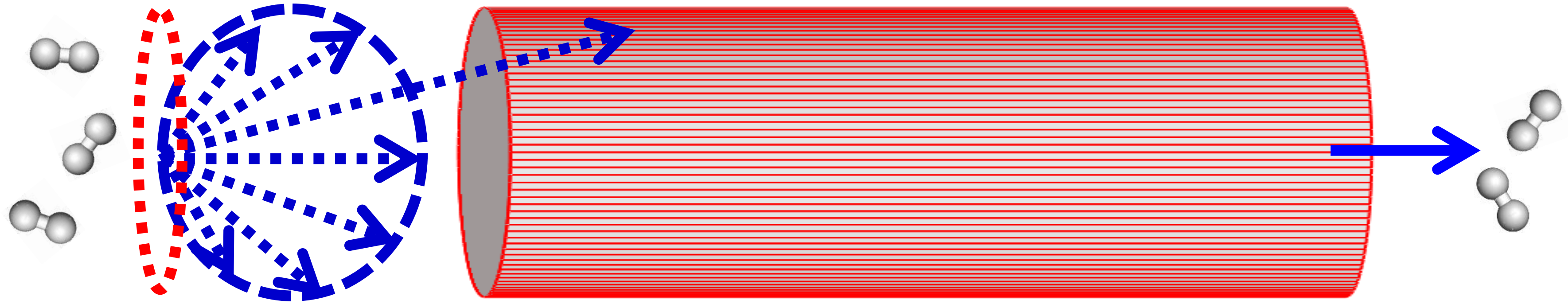


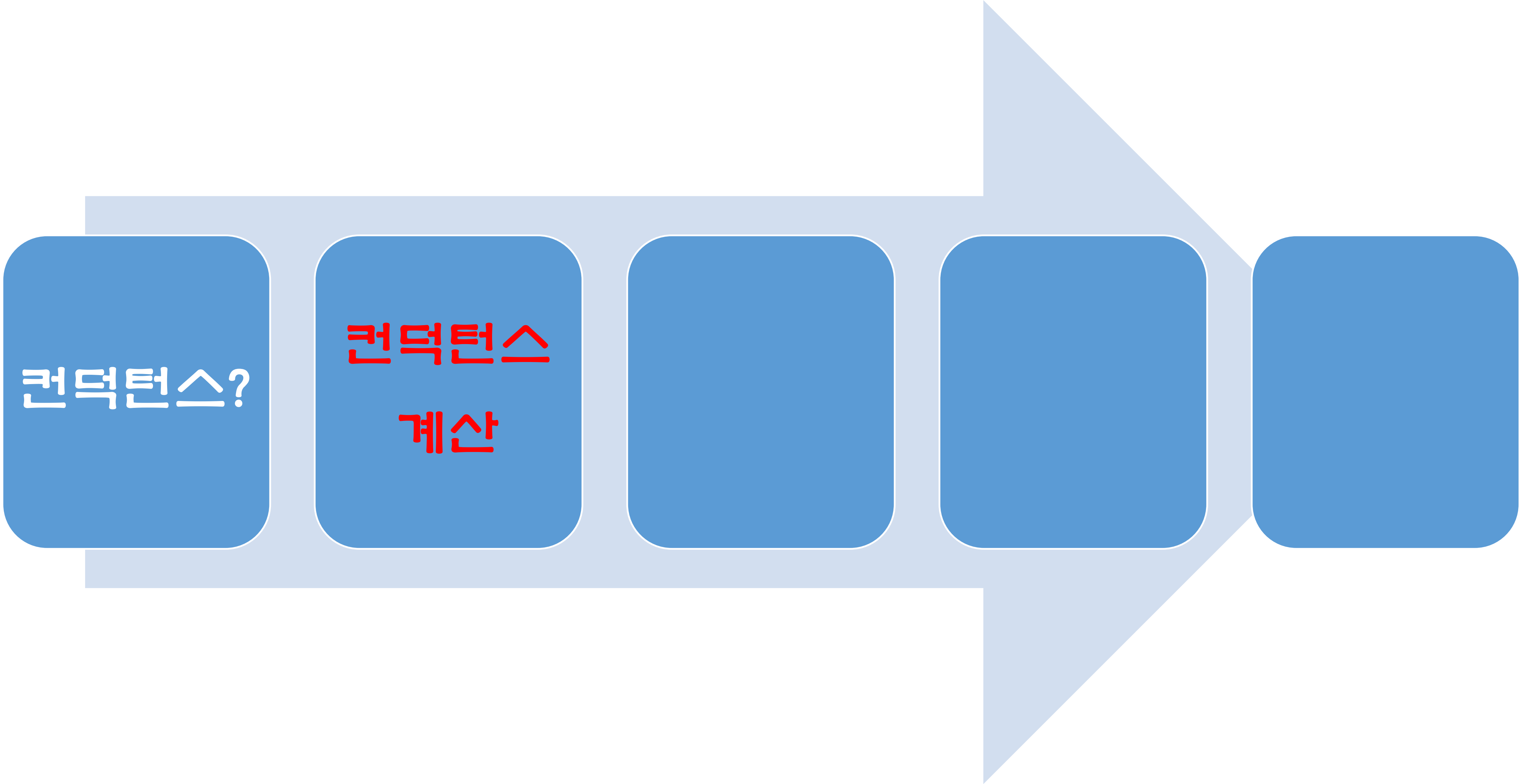
Clausing

· 입구를 찾을 확률(면적)

· 통과해 나갈 확률

$C = \alpha C_o$



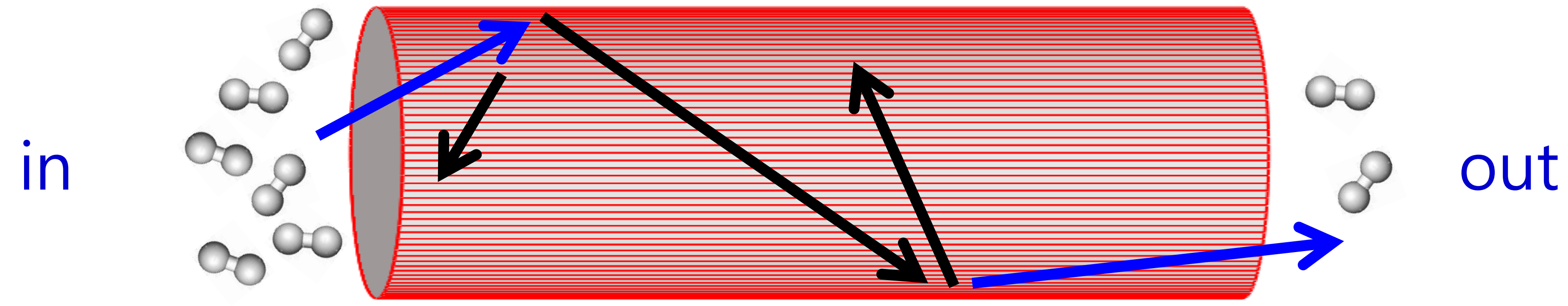

컨덕턴스?
컨덕턴스 계산

컨덕턴스 계산

- (Test Particle) Monte Carlo

- 근사 해석식

I. TPMC 계산



$$\alpha = \frac{\#\ \mathrm{out}}{\#\ in}$$

예제) TPMC 계산

- L/d=1 원형 튜브

- $\boldsymbol{\alpha} = \frac{\#\ \mathrm{out}t}{\#\ in}$

  $=0.514$

II. 근사 해석식: 원형도관의 α

- Knudsen
  - $\alpha = \frac{4}{3}\frac{d}{L}$ for a long tube (L/d>20)
  - $C = C_o\,\frac{4}{3}\frac{d}{L}$

    $= 12.1\,\frac{d^3}{L}$ L/s (단위: cm)

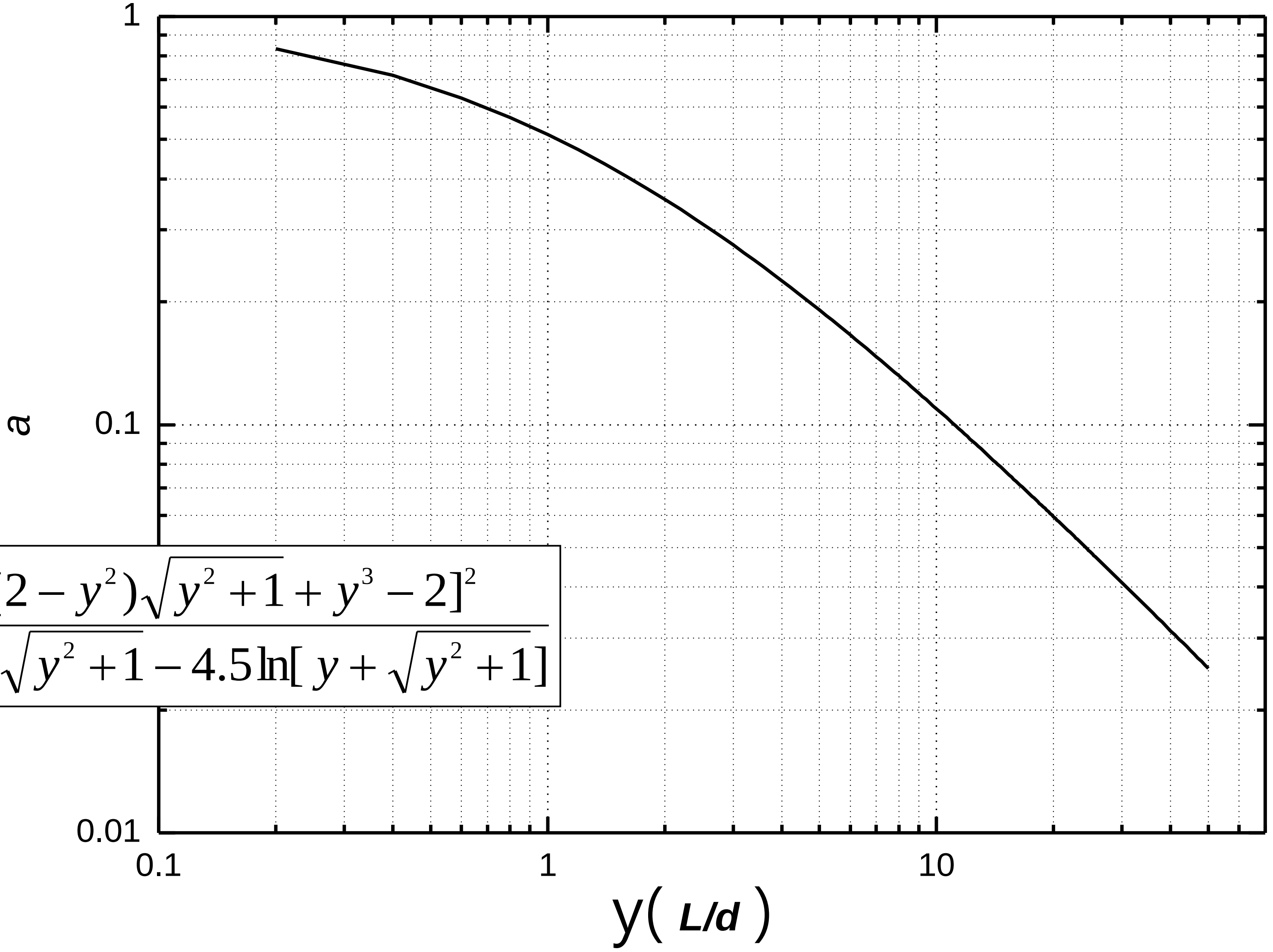
- Kennard
  - $\alpha = \frac{1}{1+\frac{L}{d}}$ $\left(\frac{1}{\alpha} = 1 + \frac{L}{d}\right)$ for a short tube (L/d<20)

II. 근사 해석식: 원형도관의 α

- Dushman
  - $\alpha = \frac{1}{1+\frac{3L}{4d}}$ $\left(\frac{1}{\alpha} = 1 + \frac{3}{4}\frac{L}{d}\right)$
  - Error<12%

- Santelar
  - $\alpha = \frac{1}{1+\frac{3L'}{4d}}$ $\left[\frac{1}{\alpha} = 1 + \left(\frac{4}{3}\frac{d}{L'}\right)^{-1}\right]$
    - $L' = \left(1 + \frac{1}{3+\frac{6L'}{7d}}\right)L$
  - Error<0.7%

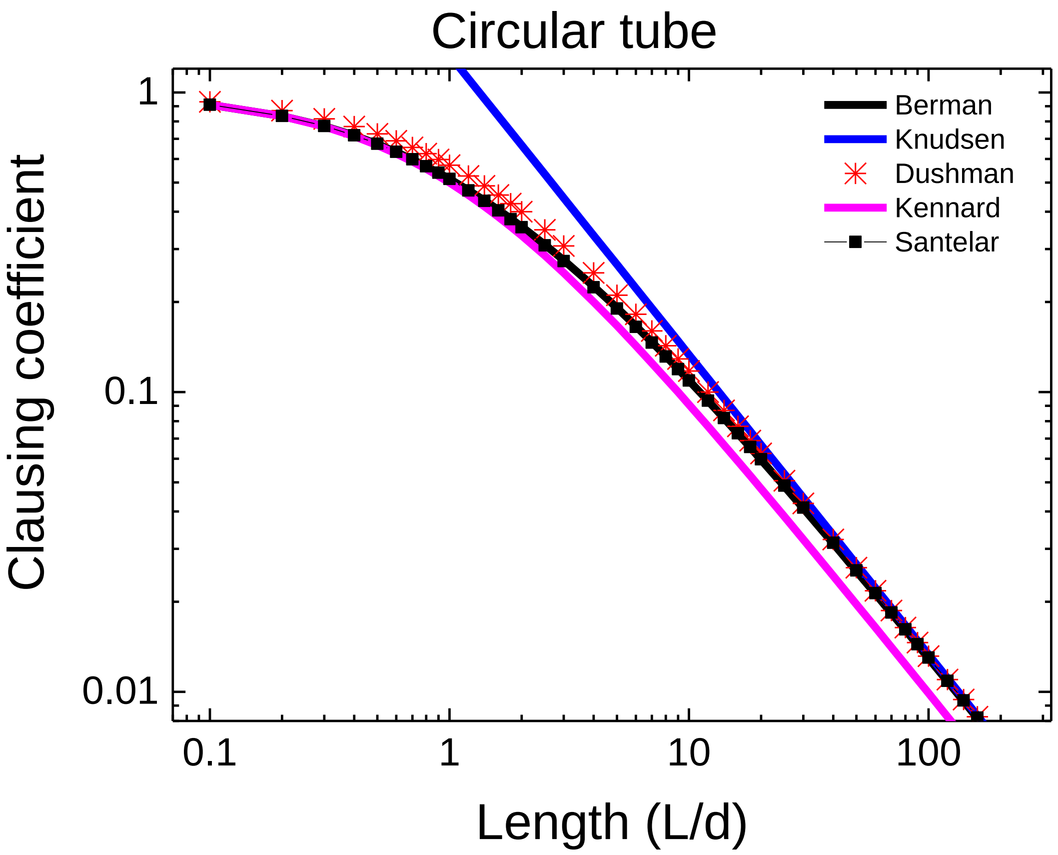
- Berman



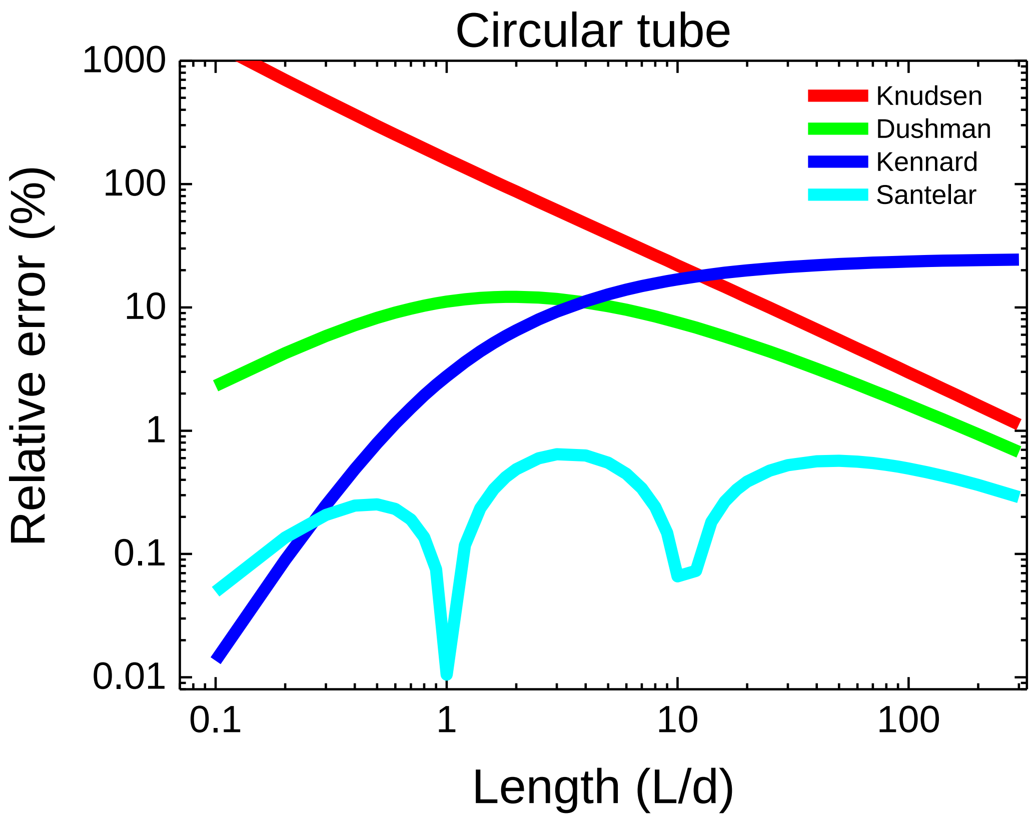
$$a = 1 + y^2 - y\sqrt{y^2+1} - \frac{[(2-y^2)\sqrt{y^2+1} + y^3 - 2]^2}{4.5y\sqrt{y^2+1} - 4.5\ln[y + \sqrt{y^2+1}]}$$

Error<0.7%

비교



Circular tube
Berman
Knudsen
Dushman
Kennard
Santelar
1
0.1
0.01
0.1
1
10
100
Clausing coefficient
Length (L/d)



Circular tube
Knudsen
Dushman
Kennard
Santelar
1000
100
10
1
0.1
0.01
0.1
1
10
100
Relative error (%)
Length (L/d)

α 를 구했으면 컨덕턴스는

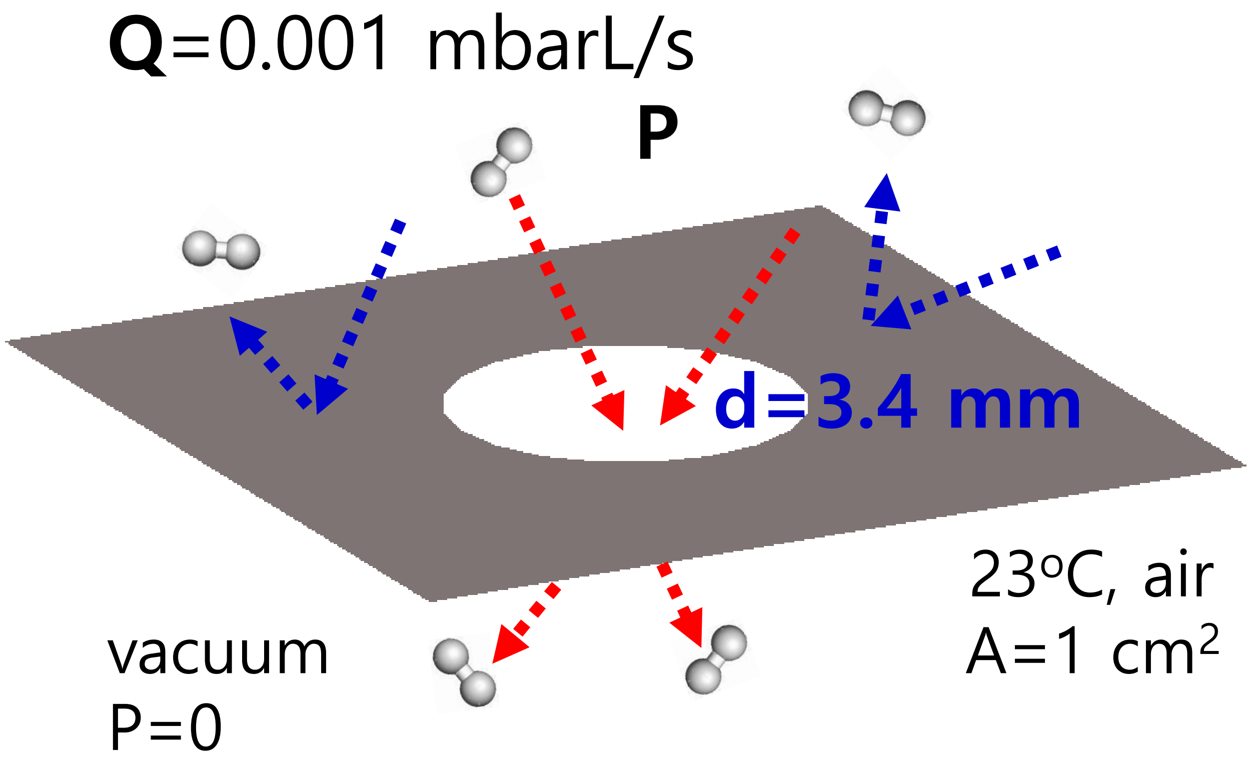
$C_{air} = \beta \; 11.6 \; \alpha \; A \quad L/s$

(23°C, air, cm)

	질량(M)	공기에 대한 비율 β
수소(H_2)	2	3.81
헬륨(He)	4	2.69
질소(N_2)	28	1.02
산소(O_2)	32	0.952
알곤(Ar)	40	0.851

$\beta = \sqrt{\frac{29}{M}}$

예제 (오리피스)



$$C = 11.6\ \color{red}{(1)}\ A$$

$$= 11.6\ (3.14 \times 0.17^2)$$

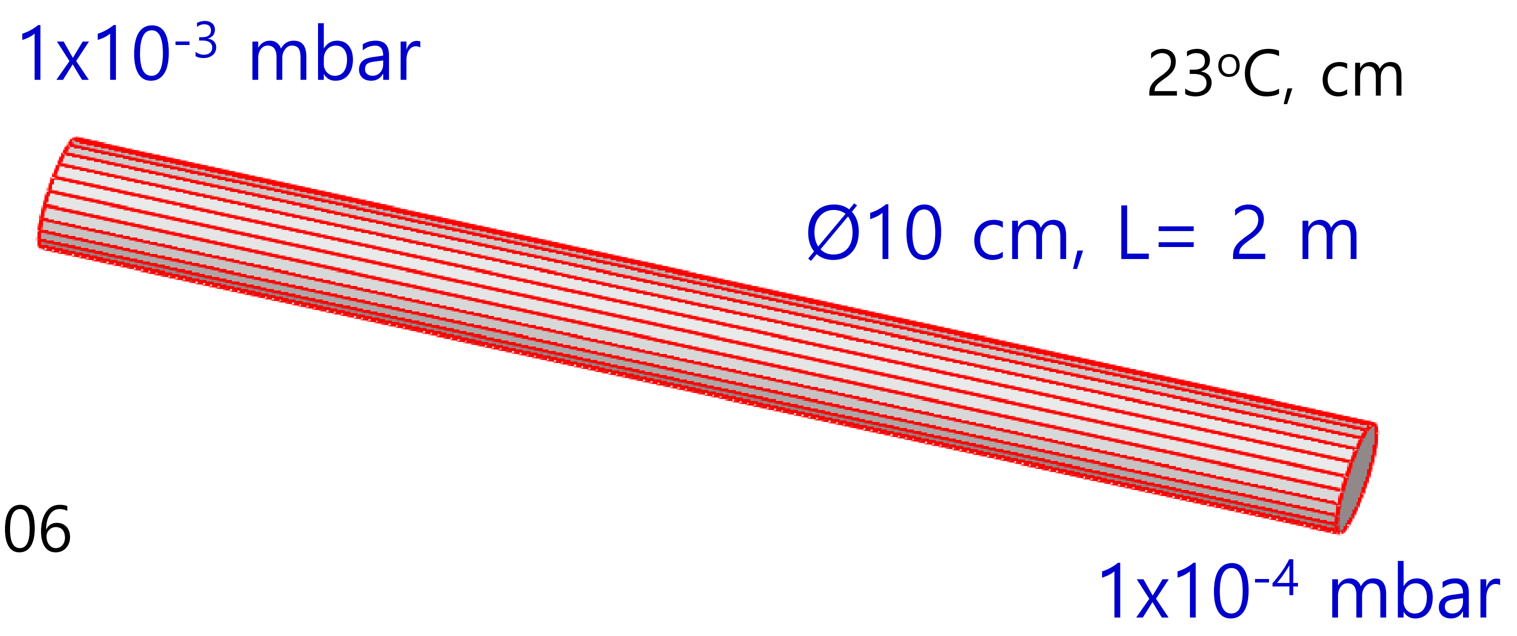
$$= 1.06\ \text{L/s}$$

$$P = Q/C$$

$$= 0.001/1.06$$

$$\sim 0.001\ \text{mbar}$$

예제 (튜브)



1. α (L/d = 200/10 = 20) = 0.06

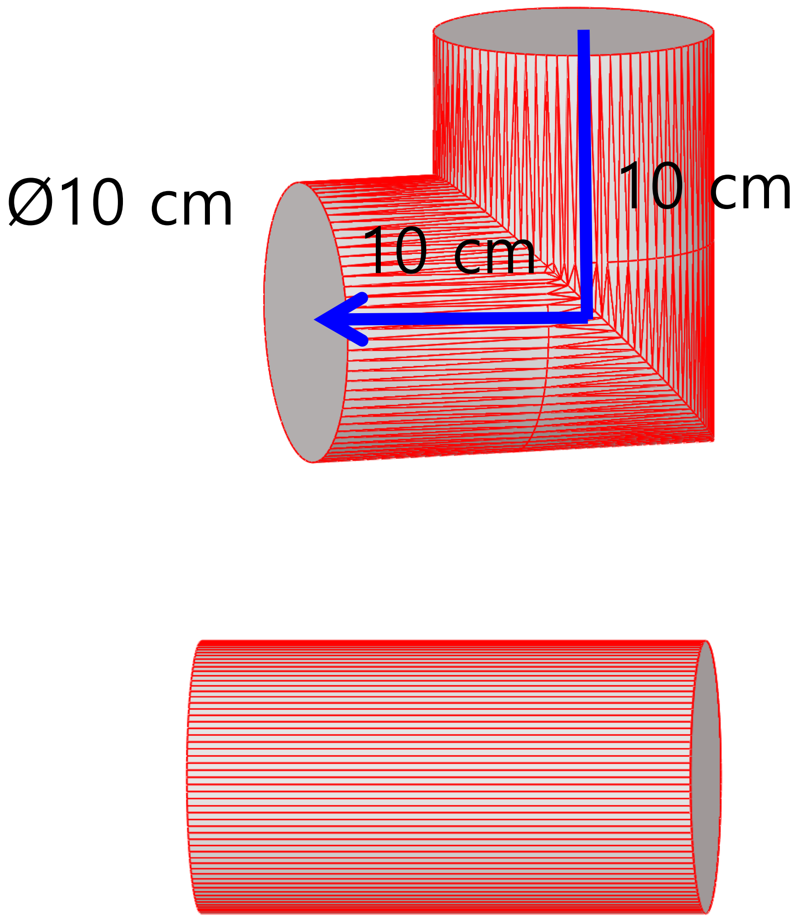
   * Dushman $\alpha = \frac{1}{1+\frac{3L}{4d}}$ = 0.062

3. C_{air} = α (11.6 π 5^2) = 57.1 L/s　　MC: 54.2 L/s (5.3% error)

   C_{H2} = 3.81 x C_{air} = 217 L/s

4. 유량 Q_{air} =C (ΔP)= 0.05 mbar·L/s

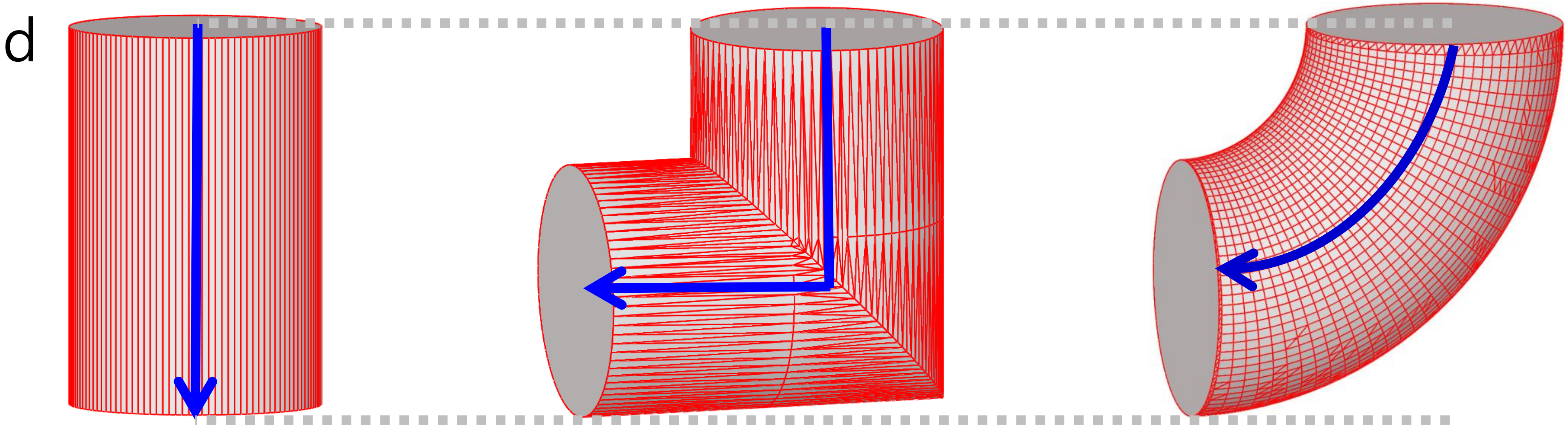
예제 (엘보)



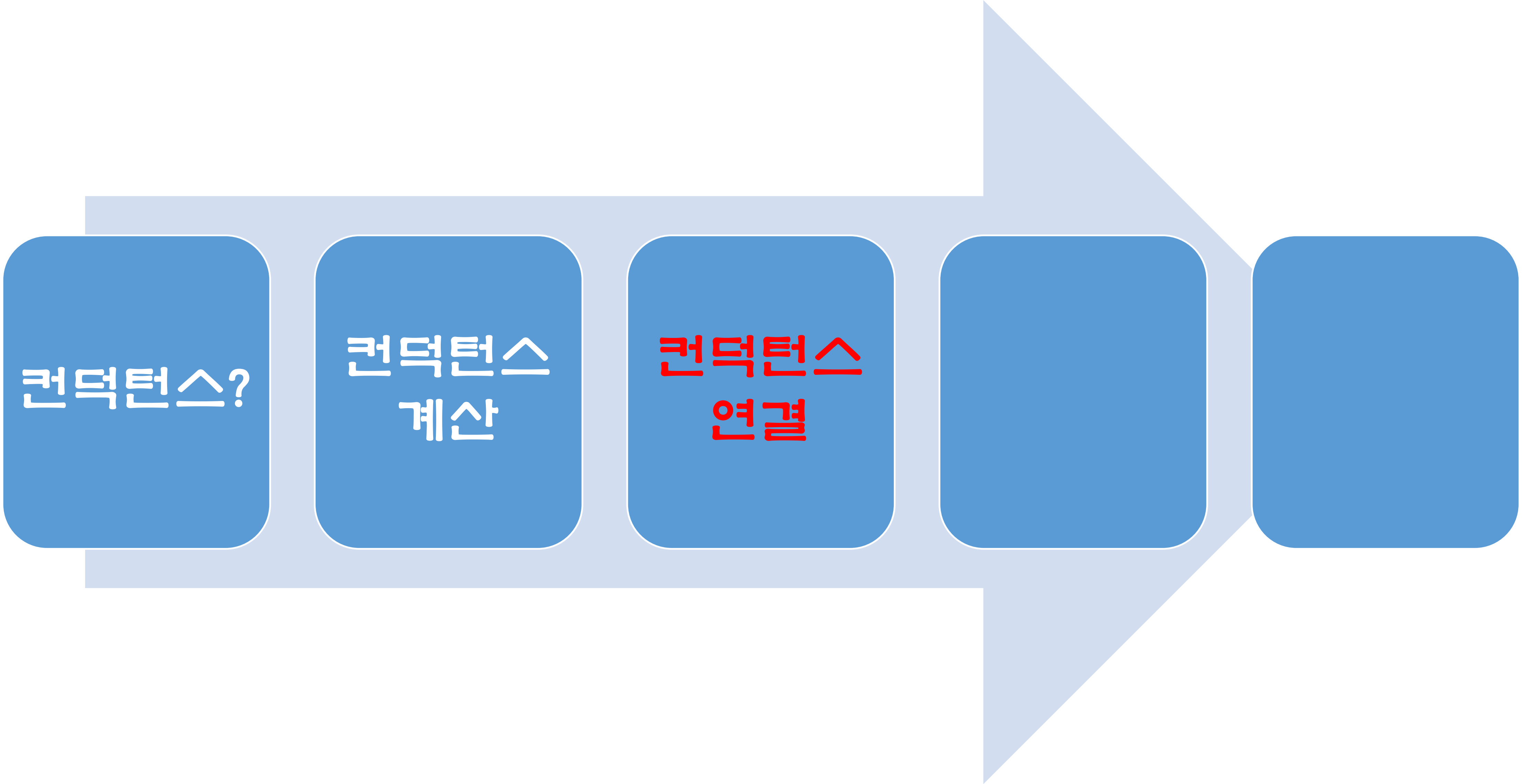
* 23°C, 공기 (cm)

$C=11.6\alpha A$
$=11.6(0.356)(3.14\text{x}5^2)$
$=324$ L/s

예제 (엘보)



L/d=	1.5	2	1·π/2 (=1.57)
(Berman) α=	0.42	0.357	0.41
error	0.02%	1.45%	4.45%
(TPMC) α=	0.42	0.351	0.392


컨덕턴스?
컨덕턴스 계산
컨덕턴스 연결

컨덕턴스 연결

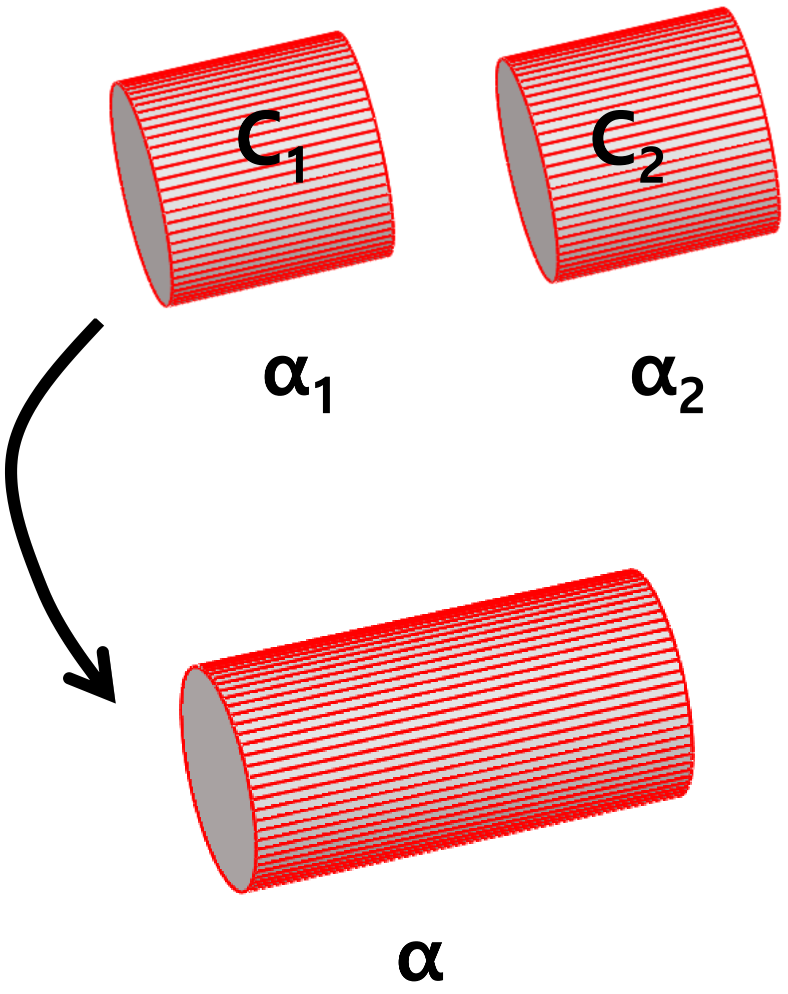
(I) 전체 길이로 계산

(II) 연결식 사용

$$\frac{1}{\alpha} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

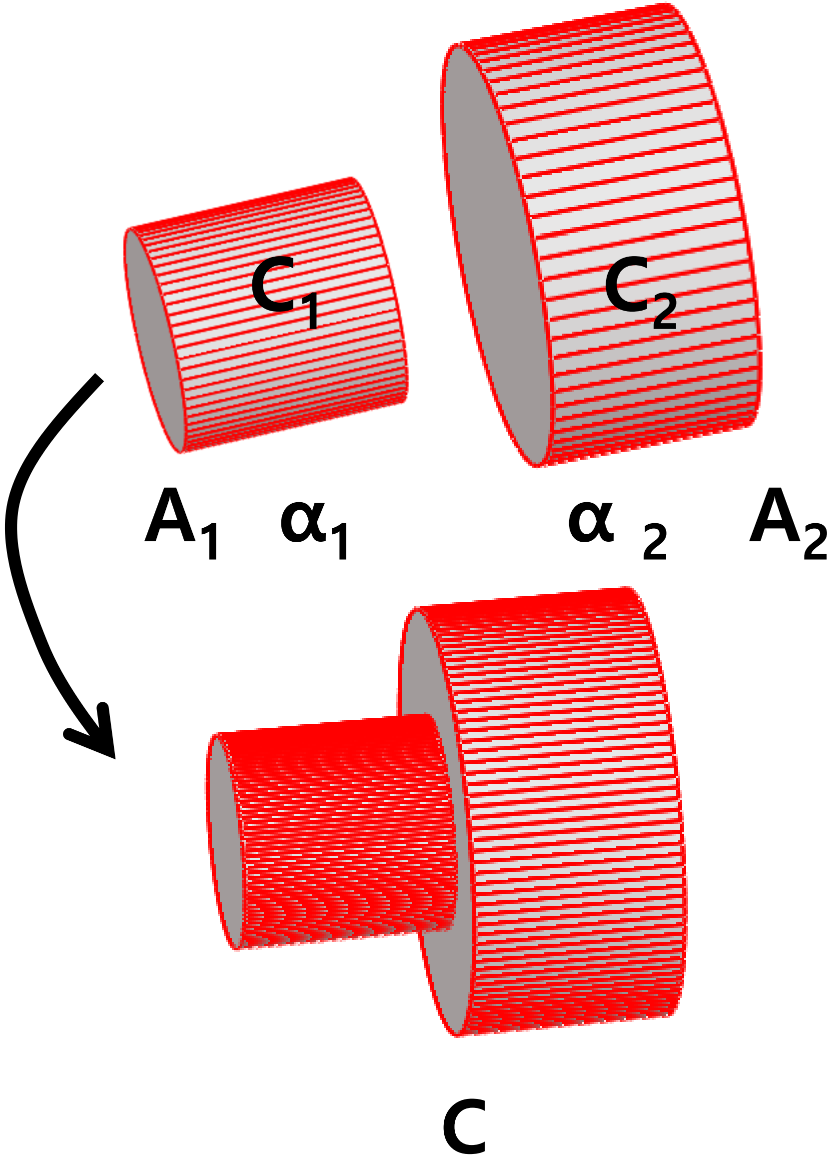
예제 (전체 길이로 …)



• 전체 길이로

$\alpha = 0.357$ (L/d=2)

예제 (전체 길이로 …)



- 전체 길이로

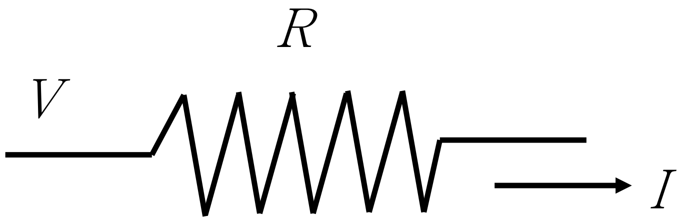
  $\alpha = 0.511$ (L/d=1&2)

II. 연결식 사용

- 전기회로와 비슷하므로 옴의 법칙 사용

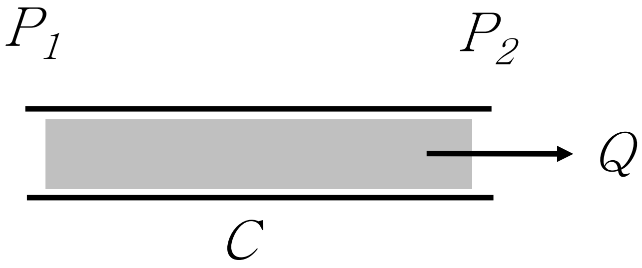
- **Current flow**

  I = (1/R) V



- **Mass flow**

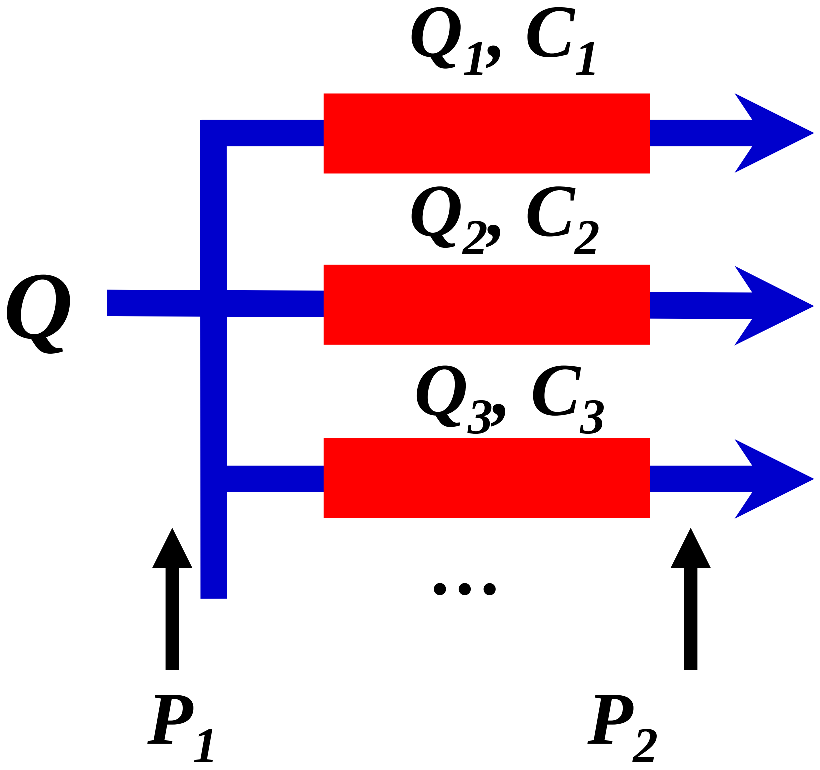
  Q = C∆P



병렬 연결

$$Q = C\Delta P = C(P_1 - P_2) = Q_1 + Q_2 + Q_3 \ \ldots = C_1\Delta P + C_2\Delta P + C_3\Delta P \ \ldots$$

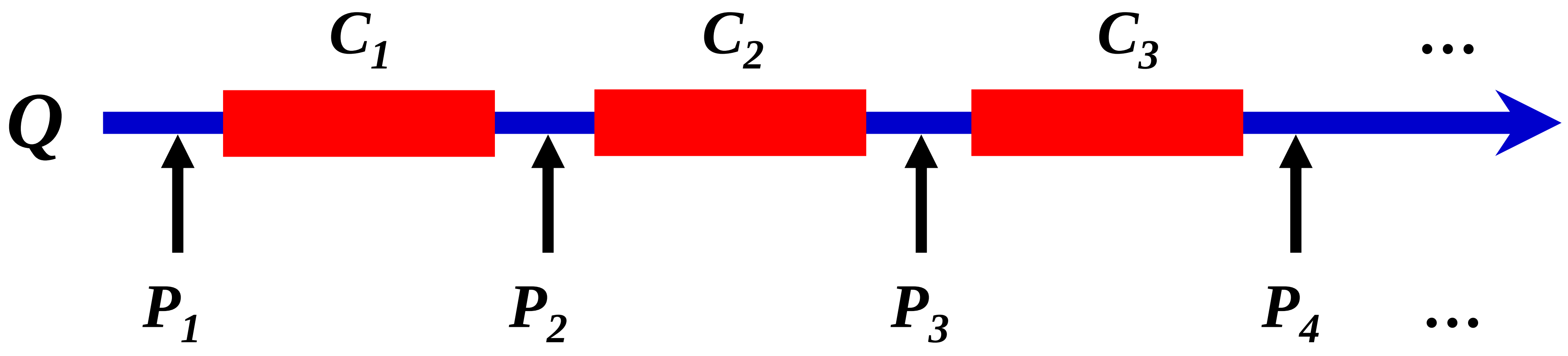
$$C = \Sigma C_i$$



직렬 연결

$$\Delta P = (P_1 - P_2) + (P_2 - P_3) + (P_3 - P_4) + \ldots = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \ \ldots$$

$$\frac{1}{C} = \sum \frac{1}{C_i}$$



C vs $\sum C_i$: Errors (옴법칙 사용)

- N >>

$$\frac{1}{\alpha} = \frac{N}{\alpha_1} \leftarrow [\frac{1}{\alpha_1} = 1 + (\frac{L/N}{d})]$$

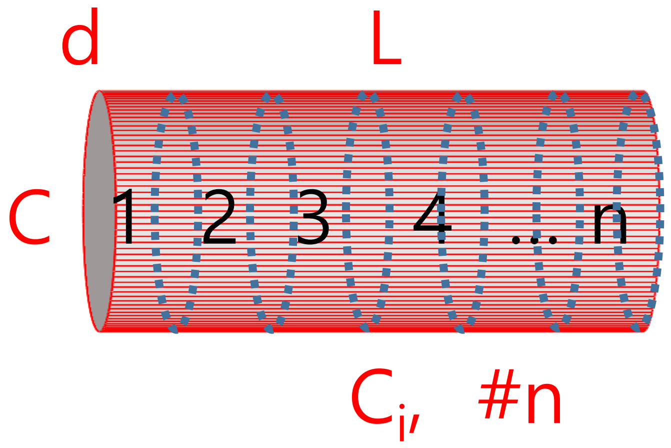
$$= N + \frac{L}{d}$$

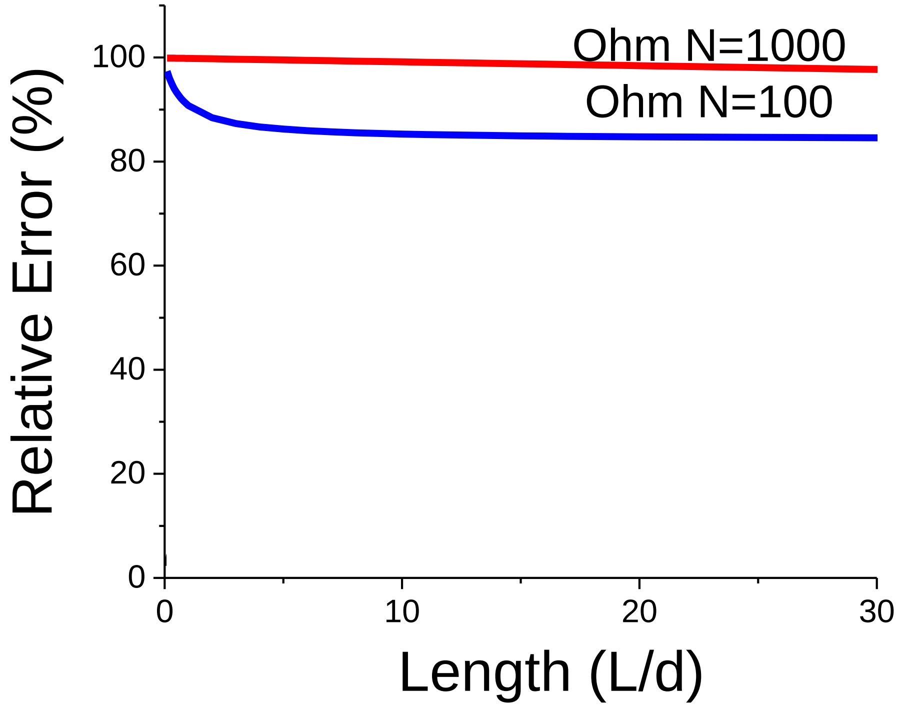
- N=1, Long

$$\frac{1}{\alpha} = 1 + \frac{3}{4}\frac{L}{d}$$

- **Error**

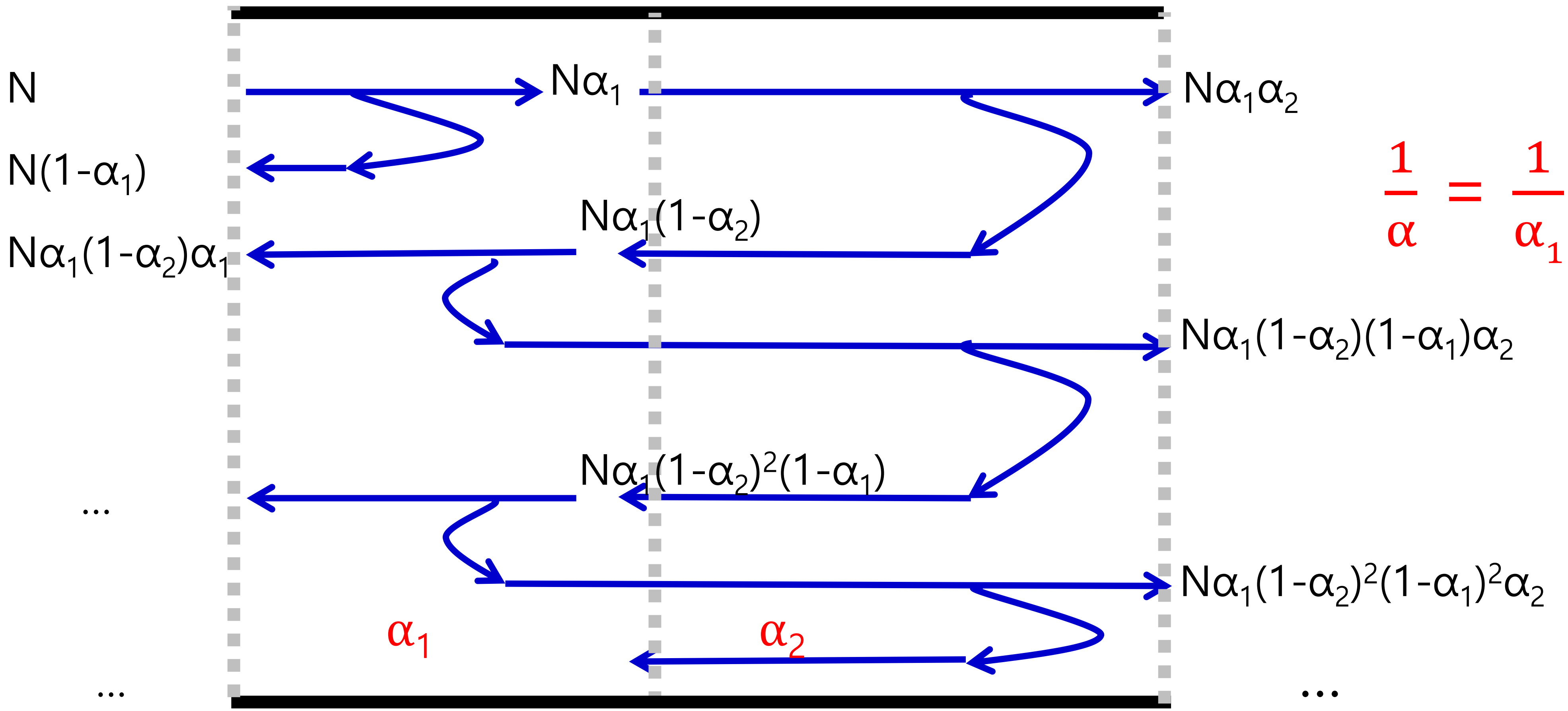
$$(N + \frac{L}{d})/(1 + \frac{3}{4}\frac{L}{d})$$





연결방법 개선: Oatley

$$\alpha = N_{out}/N = \alpha_1\alpha_2[1+(1-\alpha_1)(1-\alpha_2)+(1-\alpha_1)^2(1-\alpha_2)^2+\ldots]$$



$$\frac{1}{\alpha} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} - 1$$

C vs Σ C_i : Errors (Oatley)

- N >>

$$\frac{1}{\alpha} = \frac{N}{\alpha_1} - \frac{(N-1)}{1} \leftarrow [\frac{1}{\alpha_1} = 1 + (\frac{L/N}{d})]$$

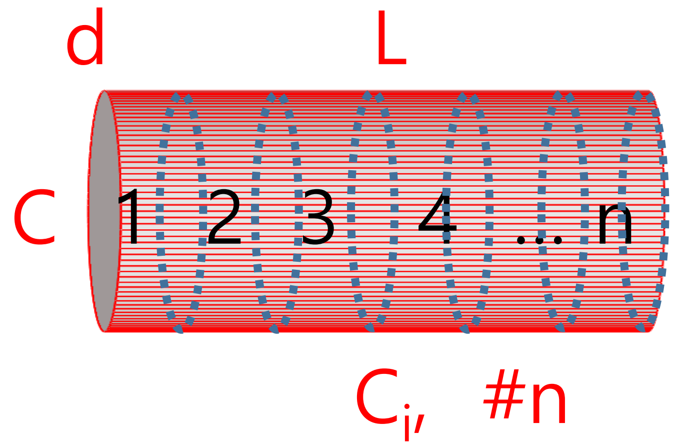
$$= 1 + \frac{L}{d}$$

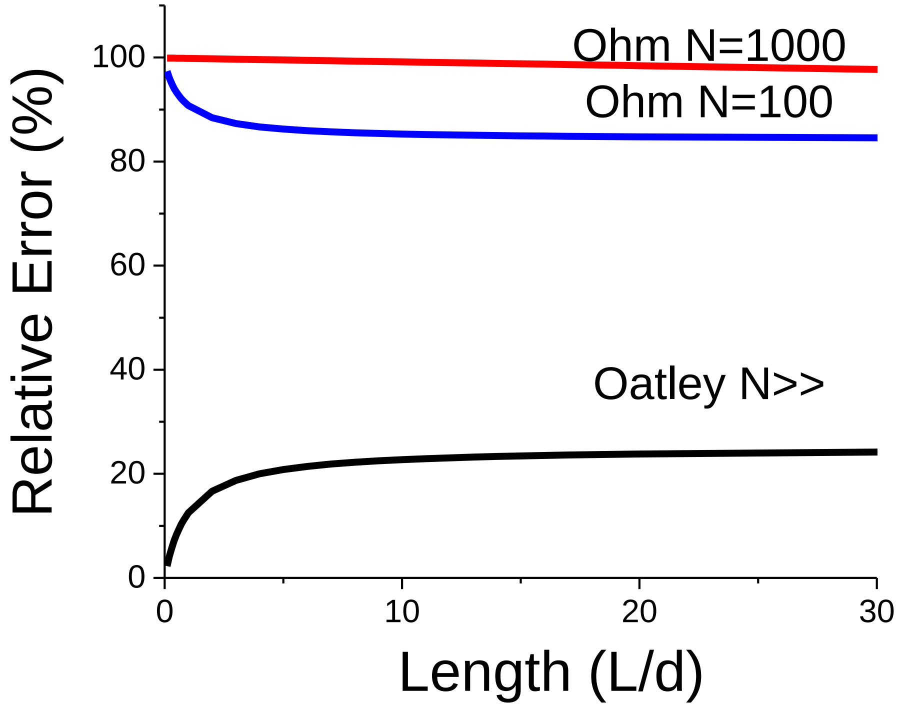
- N=1, Long

$$\frac{1}{\alpha} = 1 + \frac{3}{4}\frac{L}{d}$$

- **Error**

$(1+\frac{L}{d})/(1 + \frac{3}{4}\frac{L}{d})$ **<25%**





컨덕턴스 연결: 오차 줄이기

I. 전체 길이에 대한 통과확률 적용

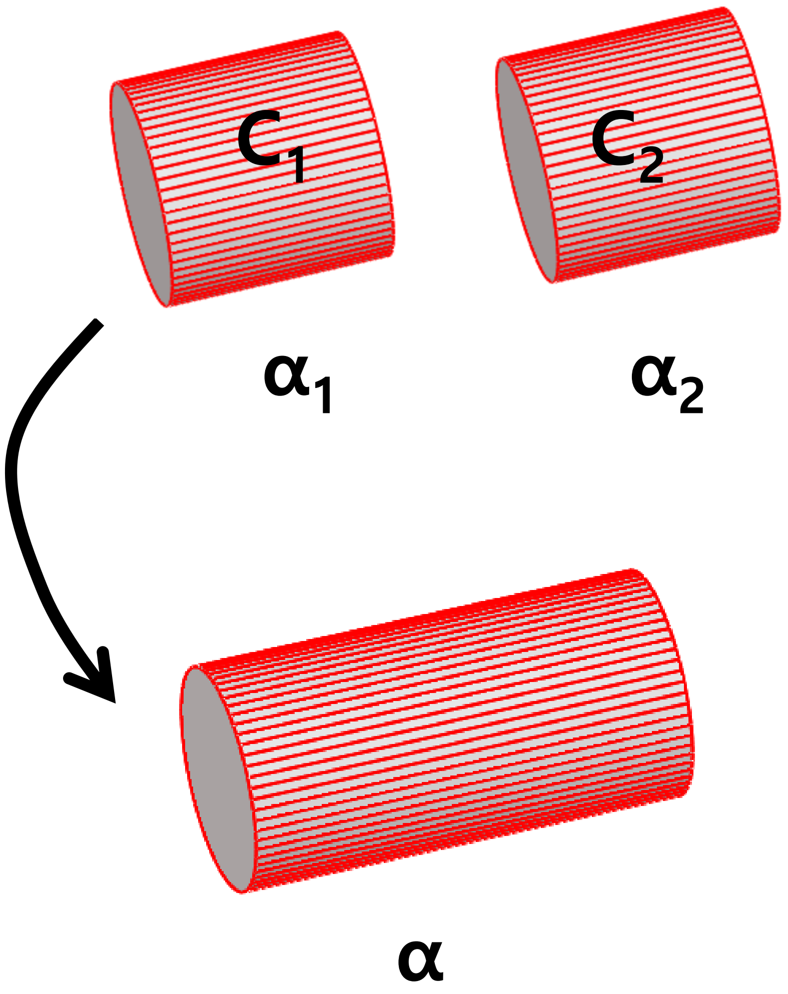
II. 연결식 사용: Oatley

$$\frac{1}{\alpha} = \frac{1}{\alpha_1} + \frac{1}{\alpha_2} - 1 \qquad \frac{1}{C} = \frac{1}{C_1} + \left(\frac{1}{C_2} - \frac{1}{C_o}\right)$$

- 좀더 정확한 연결식: Santeler

$$\frac{1}{\alpha} = \frac{1}{\alpha_1} + \left(\frac{1}{\alpha_2} - 1\right) + 0.3 \ln (\alpha_1 + \alpha_2 - \alpha_1\alpha_2)$$

예제: 같은 연결 단면



- 전체 길이로

  α= 0.357 (L/d=2)

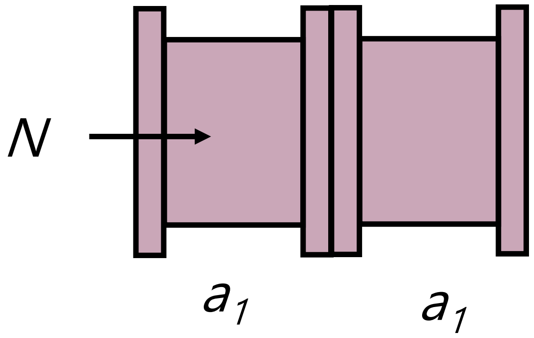
- 연결: Ohm

  $\frac{1}{\alpha} = \frac{2}{\alpha_1}$, α=0.257

- 연결: Oatley

  $\frac{1}{\alpha} = \frac{2}{\alpha_1}$ -1, α=0.346

예제: 비교



- 전체 길이로

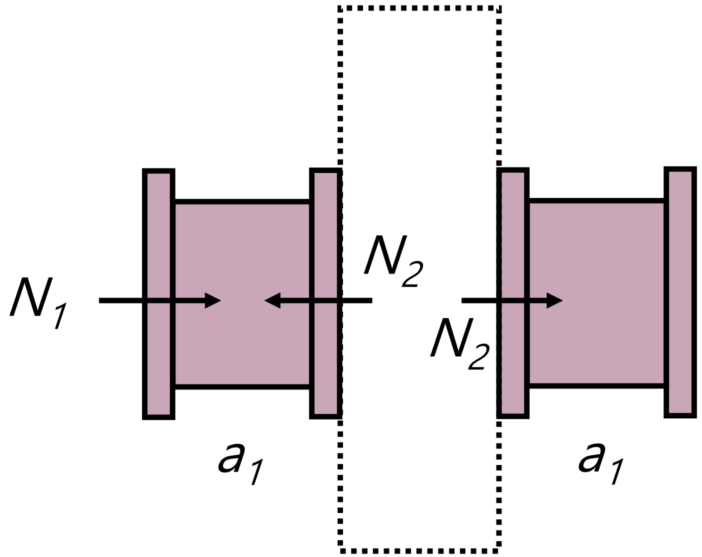
  α= 0.357 (L/d=2)

- 연결: Ohm

  $\frac{1}{\alpha} = \frac{2}{\alpha_1}$, α=0.257

- 연결: Oatley

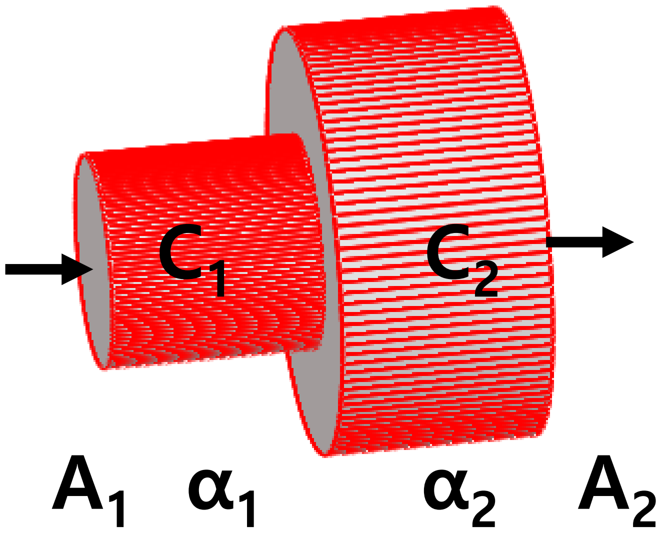
  $\frac{1}{\alpha} = \frac{2}{\alpha_1} - 1$, α=0.346



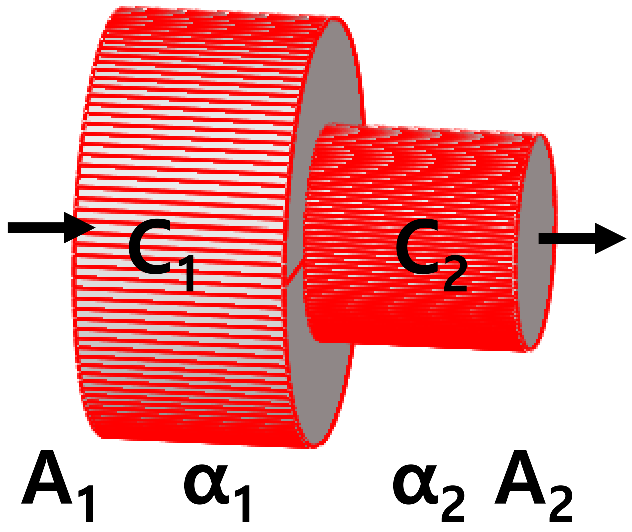
- 연결: Ohm

  $\frac{1}{\alpha} = \frac{2}{\alpha_1}$, α=0.257

예제:



$$\frac{1}{C} = \frac{1}{C_1} + \left(\frac{1}{C_2} - \frac{1}{C_{2o}} \right)$$



$$\frac{1}{C} = \frac{1}{C_1} + \left(\frac{1}{C_2} - \frac{1}{C_{1o}} \right)$$

예제:

- 원형도관 (길이 20 cm, 안지름 20 cm)을 길이 20 cm, 안지름 10 cm 인 원형도관에 연결할 때 전체 컨덕턴스는 ?

TPMC

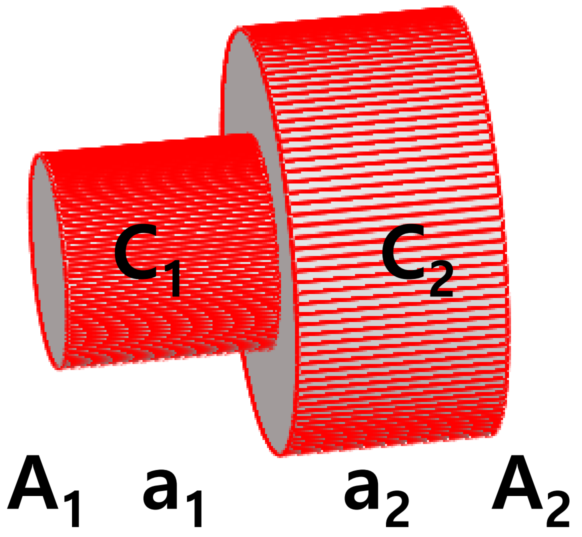
$C = 469$ L/s $[\alpha(1 \rightarrow 2) = 0.511]$

Oatley

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} - \frac{1}{C_{20}}, \quad C = 441 \text{ L/s} \quad (5.9\% \text{ error})$$

Ohm

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}, \quad C = 394 \text{ L/s } (16\% \text{ error})$$



$C_1 = 11.6aA = 469$ L/s

$C_2 = 245$ L/s

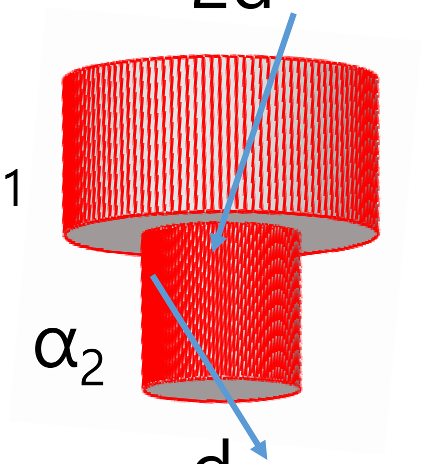
$C_{o,large}$ 365 L/s

컨덕턴스 연결; 일반식

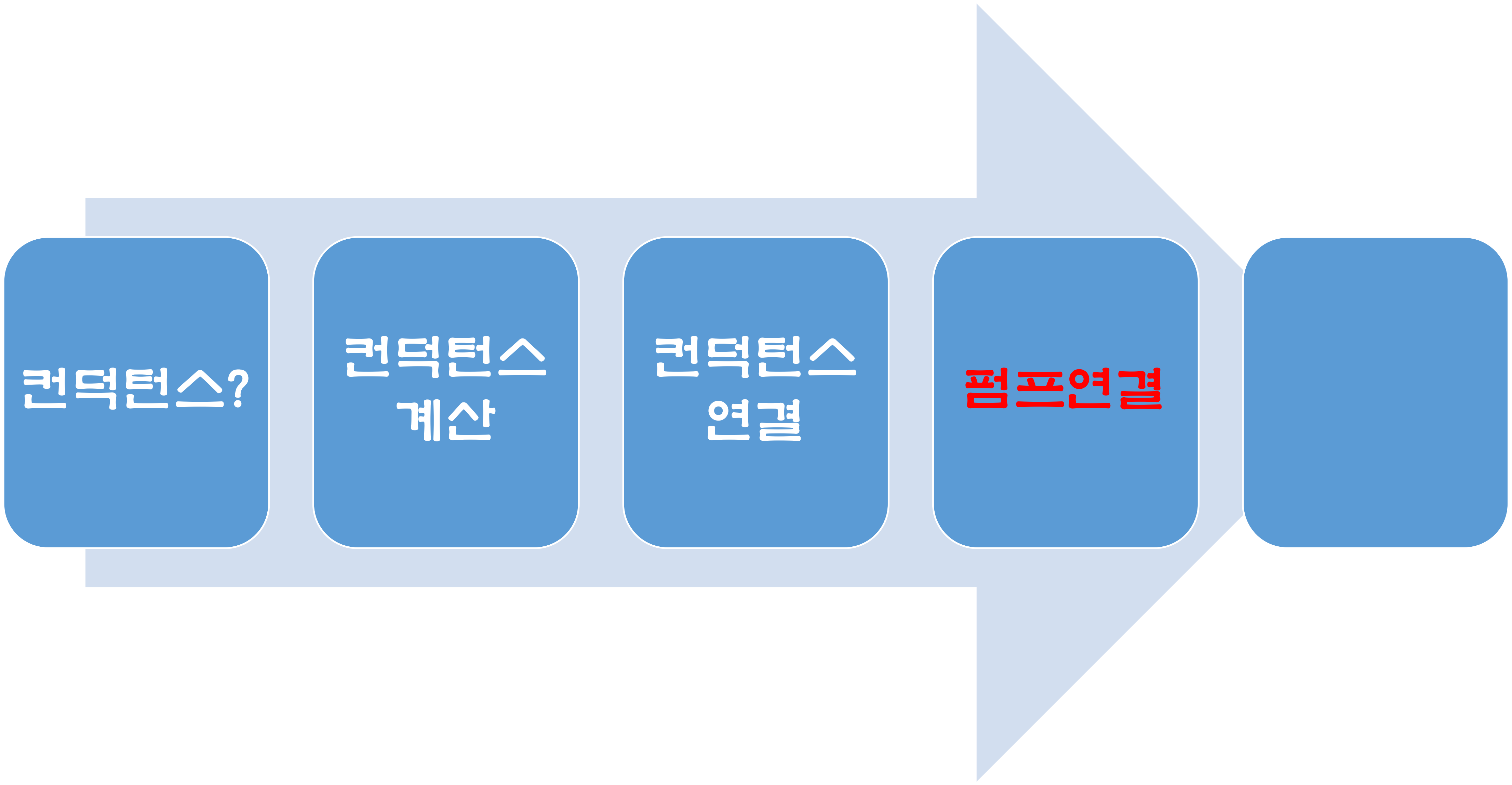
$$\frac{1}{C}=\frac{1}{C_1}+\left(\frac{1}{C_2}-\frac{1}{C_{o(\max 1|2)}}\right)+\left(\frac{1}{C_3}-\frac{1}{C_{o(\max 2|3)}}\right)+\ldots,$$

방향성

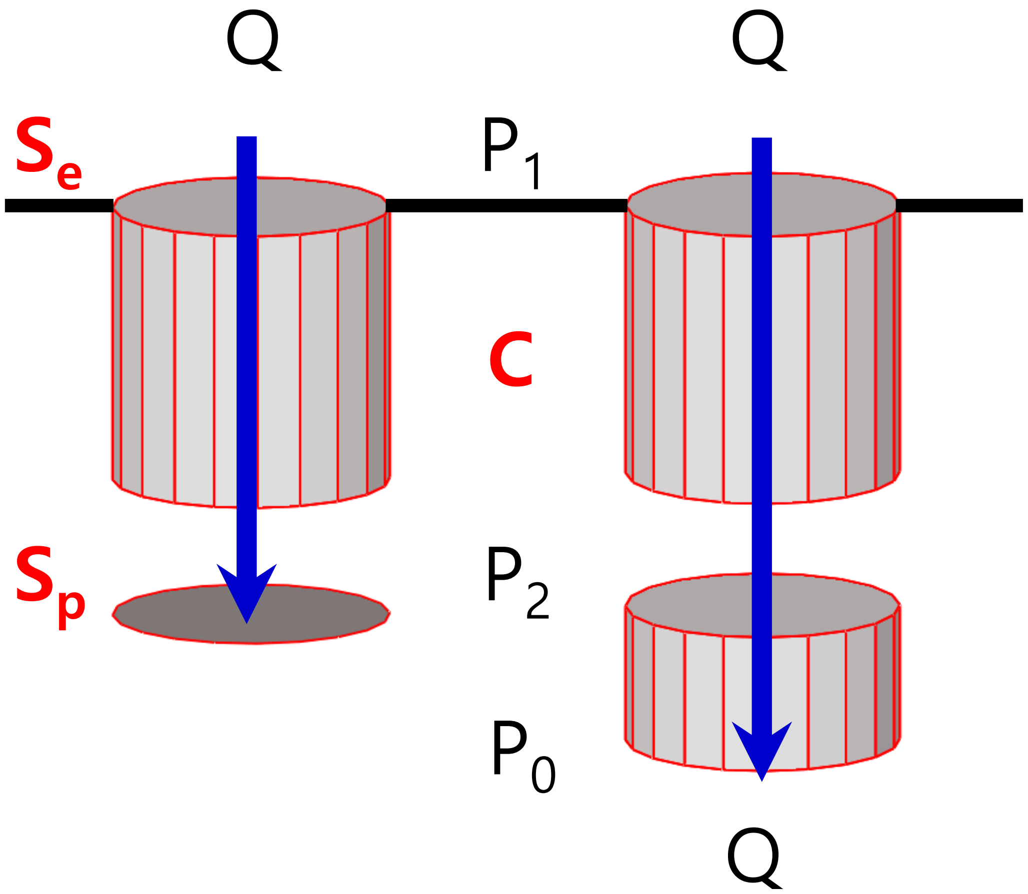
L=2d



		α	C (L/s)
$\alpha\downarrow$ =	$\frac{1}{\alpha} = \frac{1}{\alpha_1} + \left(\frac{4}{\alpha_2} - \frac{1}{1}\right)$	0.121	4.42
$\alpha\uparrow$ =	$\frac{1}{\alpha'} = \frac{1}{\alpha_2} + \left(\frac{1}{4\alpha_1} - \frac{1}{4}\right)$	0.484	4.42
			(5.9% error)
(MC) $\alpha\uparrow$		0.511	4.69


컨덕턴스?
컨덕턴스 계산
컨덕턴스 연결
펌프연결

펌프-도관 연결



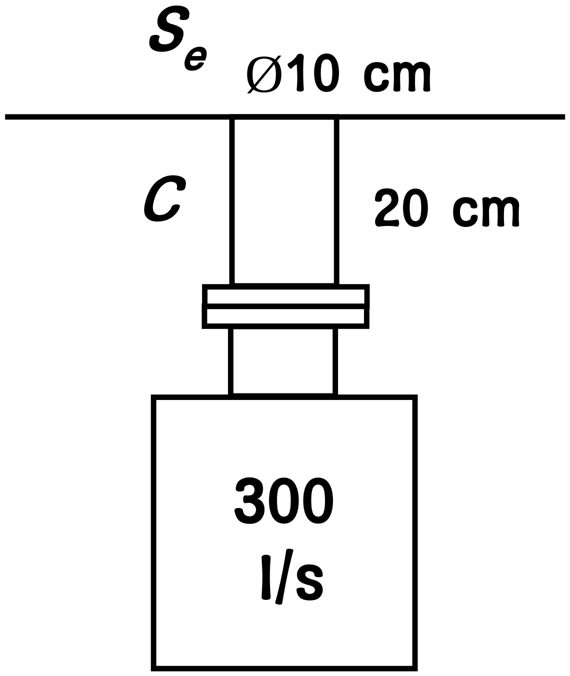
$$Q = P_1 S_e = C(P_1 - P_2) = P_2 S_p$$

$$Q = (P_1 - P_0) S_e = C(P_1 - P_2) = (P_2 - P_0) S_p$$

$$\frac{Q}{S_e} = \frac{Q}{C} + \frac{Q}{S_P}$$

$$S_p = \alpha C_o$$

예제 (유효배기속도 구하기)



* 23°C, **공기** (cm)

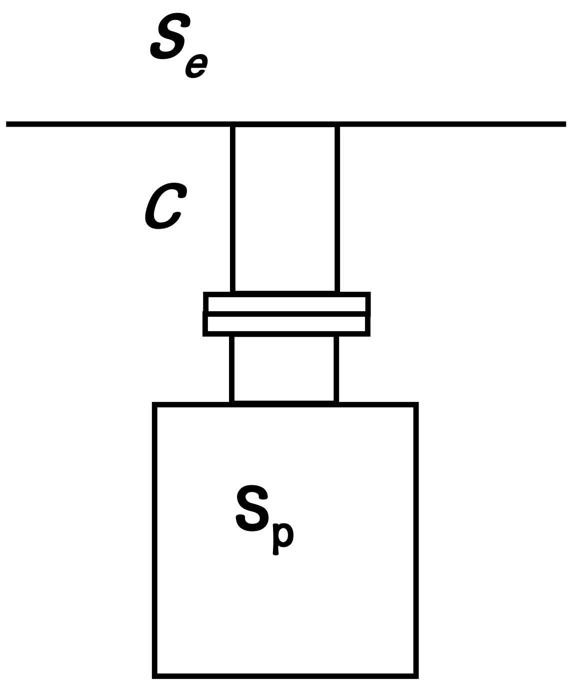
C =11.6 (0.357)(3.14x5^2)=325 l/s

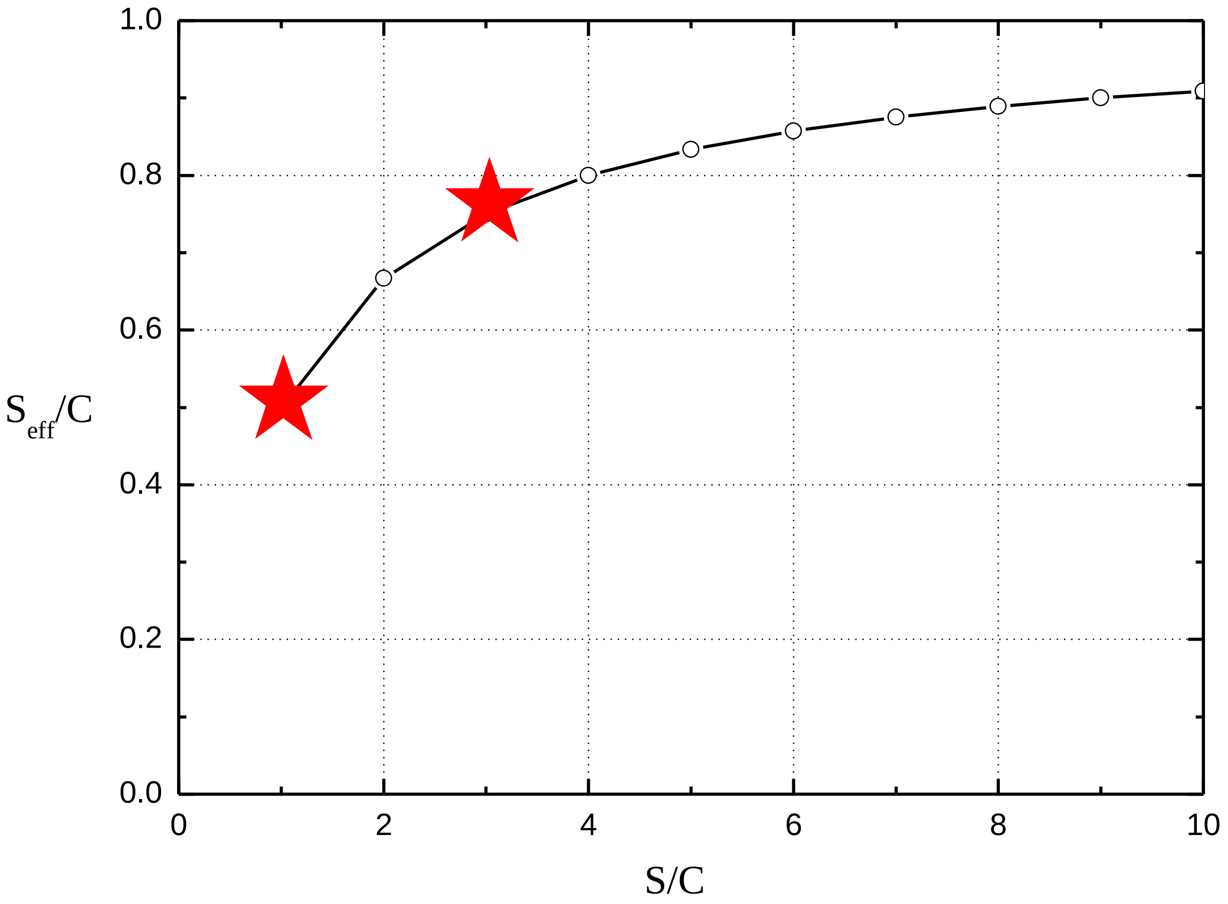
C_o=11.6 (3.14x5^2)=911 l/s

1. $$\frac{1}{S_{eff}} = \frac{1}{C} + \frac{1}{S} = 155 \quad \text{l/s}$$

2. $$\frac{1}{S_{eff}} = \frac{1}{C} + \left(\frac{1}{S} - \frac{1}{C_o}\right) = 188 \quad \text{l/s}$$

S_e





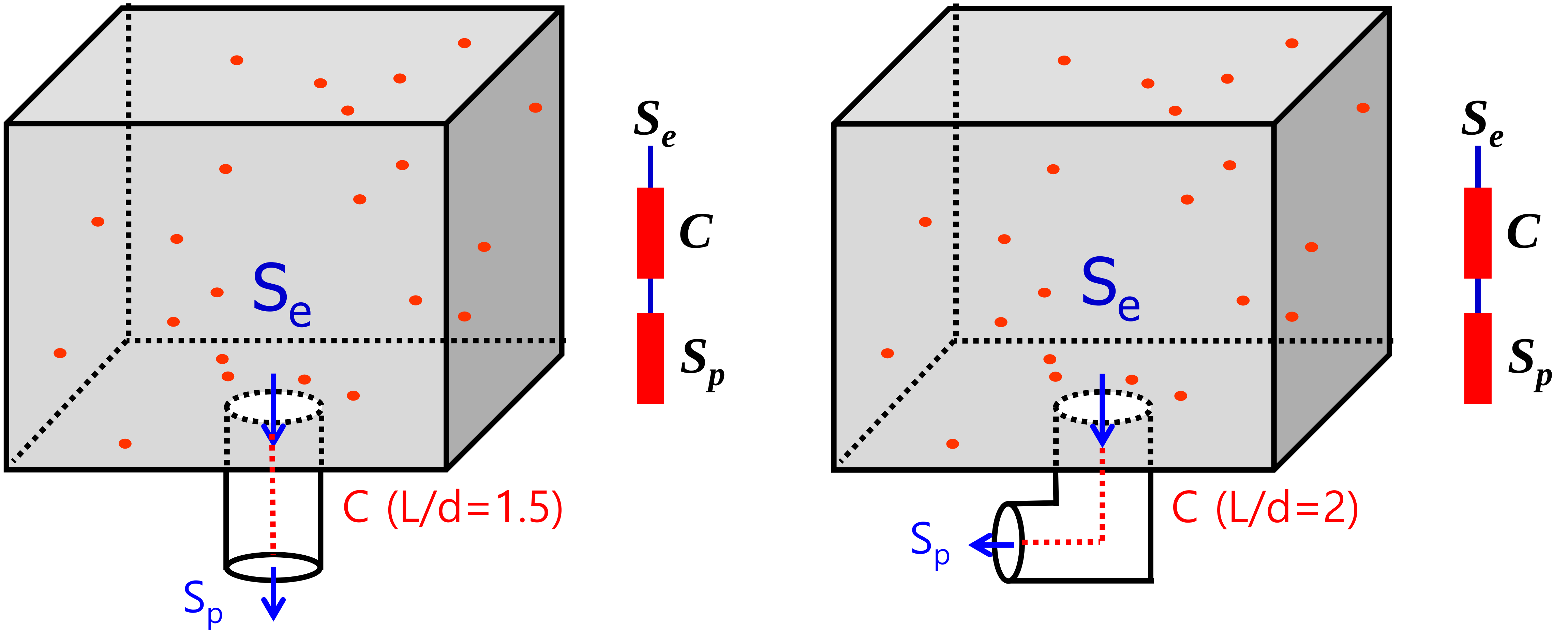
예제 (S)

- 어떤 진공 용기의 총 기체방출량이 1×10^{-7} mbarL/s(질소 등가) 이다. 이 용기에 지름 10 cm, 길이 20 cm 인 원형도관을 사용하여 고진공 펌프를 설치하고자 한다. 이 진공용기를 10^{-9} mbar로 유지하려 할 때 필요한 배기속도는?

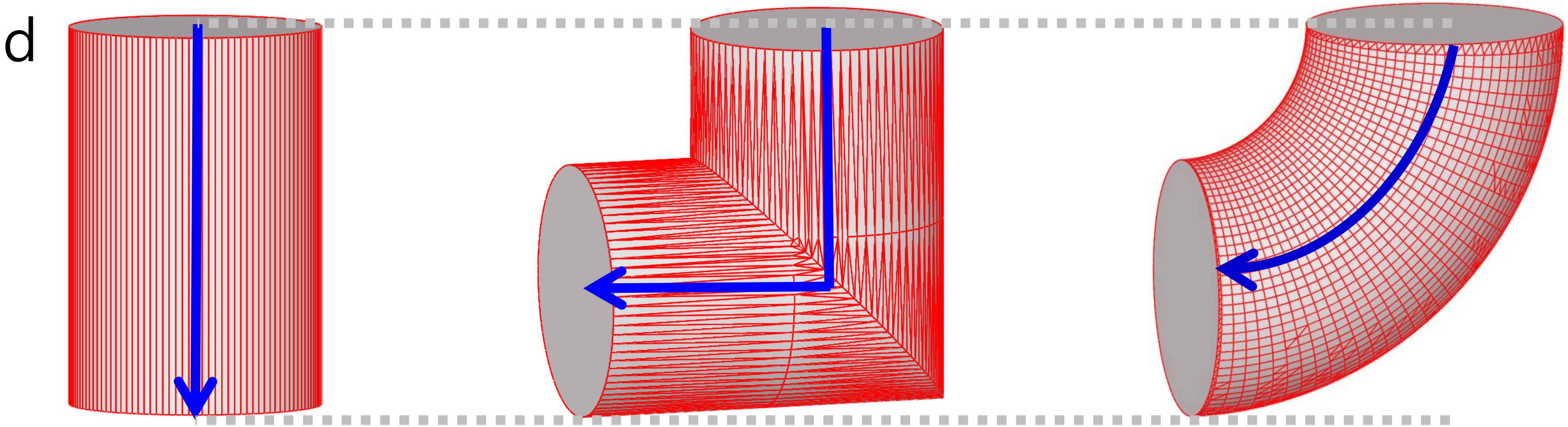
- 필요한 유효 배기속도 $S_e = \frac{Q}{P} = \frac{1e-7}{1e-9}$ = 100 L/s
- C= $11.6\alpha A$ = 11.6 (0.357) (3.14×5^2) = 325 L/s, C_o= 912 L/s
- 안지름 10 cm, 길이 20 cm인 원형도관을 사용하여야 하므로,
- $\frac{1}{S_p} = \frac{1}{S_e} + \frac{1}{C_o} - \frac{1}{C}$ → S= 125 L/s

(비교) 옴법칙으로 연결하면 $S_{pump} = \frac{CS_{eff}}{C - S_{eff}} = \frac{325 \times 100}{325 - 100} = 144 \quad (l/s)$

예제 (S_e)



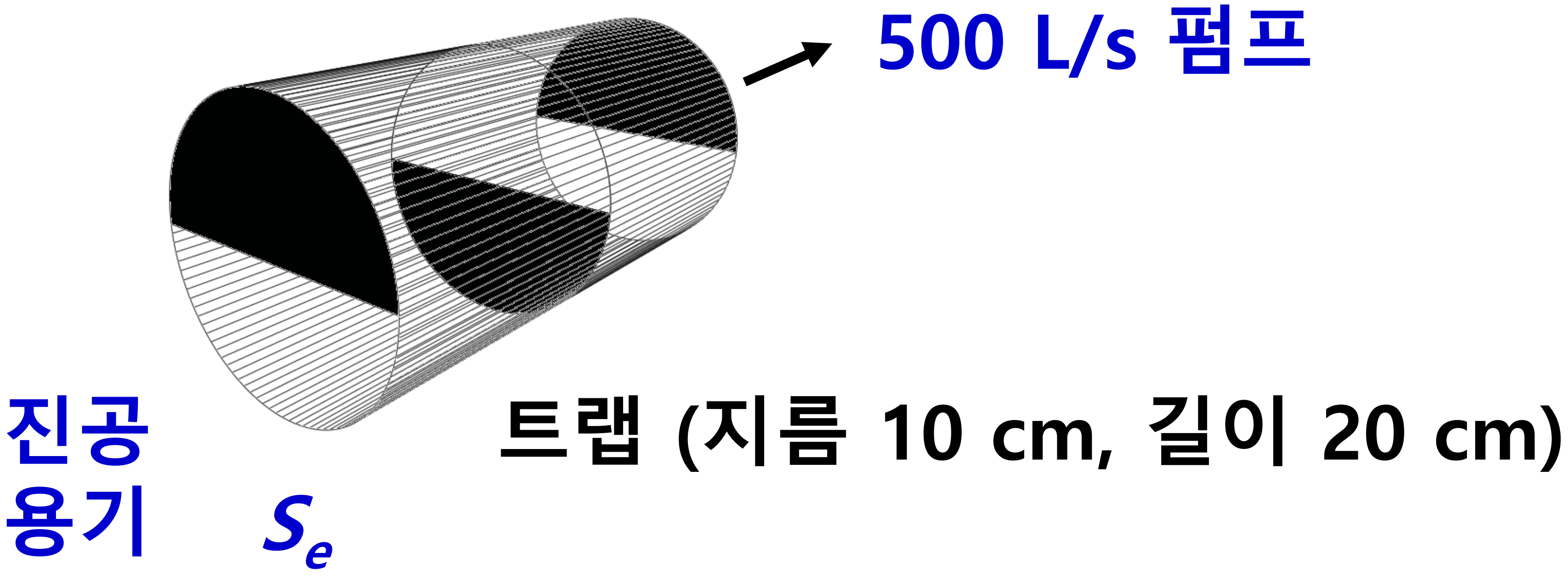
예제 S_e



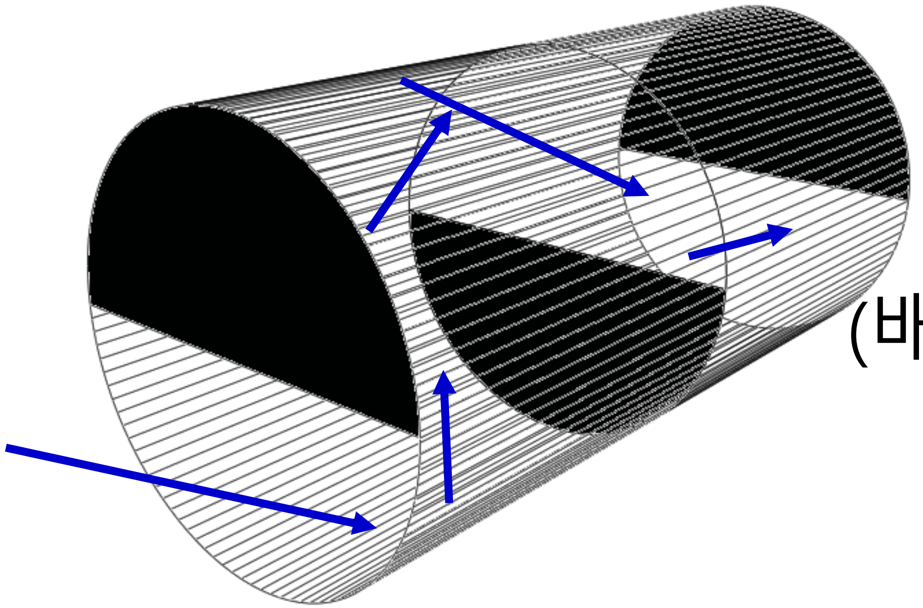
L/d=	1.5	2	1·π/2 (=1.57)
(Berman) α=	0.42	0.357	0.41
error	0.02%	1.45%	4.45%
(MC) α=	0.42	0.351	0.392

예제 (S_e)

- 어떤 진공 용기에 그림과 같은 트랩(지름 10 cm)과 고진공 펌프(500 L/s)를 붙였다고 하자. 이때 진공용기 입구에서 유효 배기속도는?



예제



A (입구단면적)

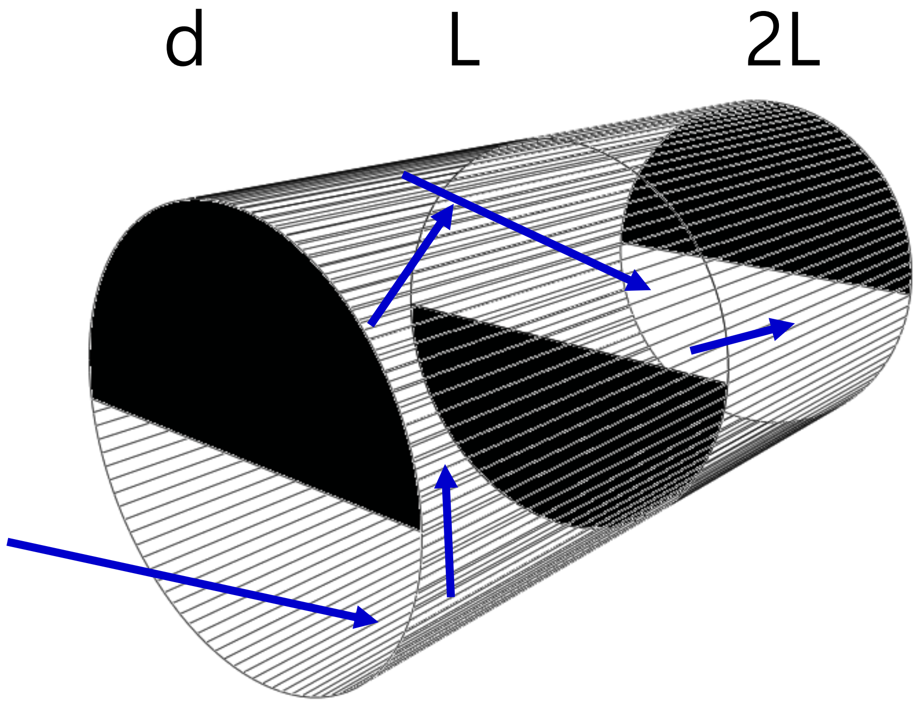
S_e (유효배기속도)

$$\frac{1}{Se} = \frac{1}{C} + \left(\frac{1}{S} - \frac{1}{C_o}\right)$$

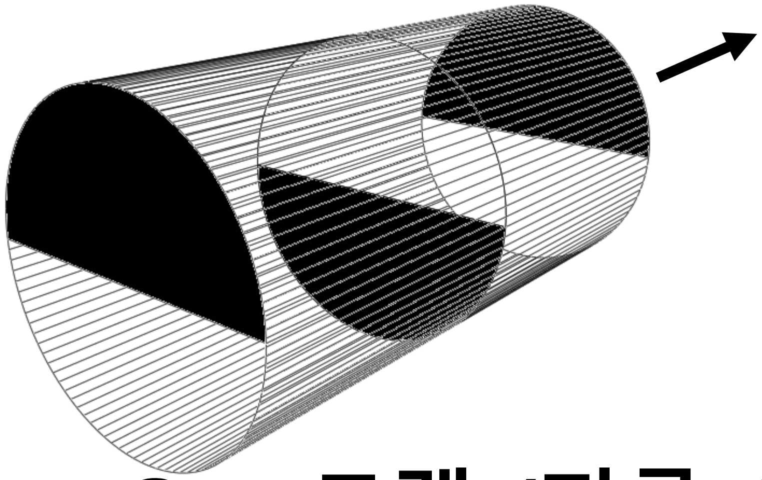
$$= \frac{1}{C} + \left(\frac{1}{500} - \frac{1}{913}\right)$$

$C_o = 11.6\ (3.14x52) = 913\ L/s$

예제

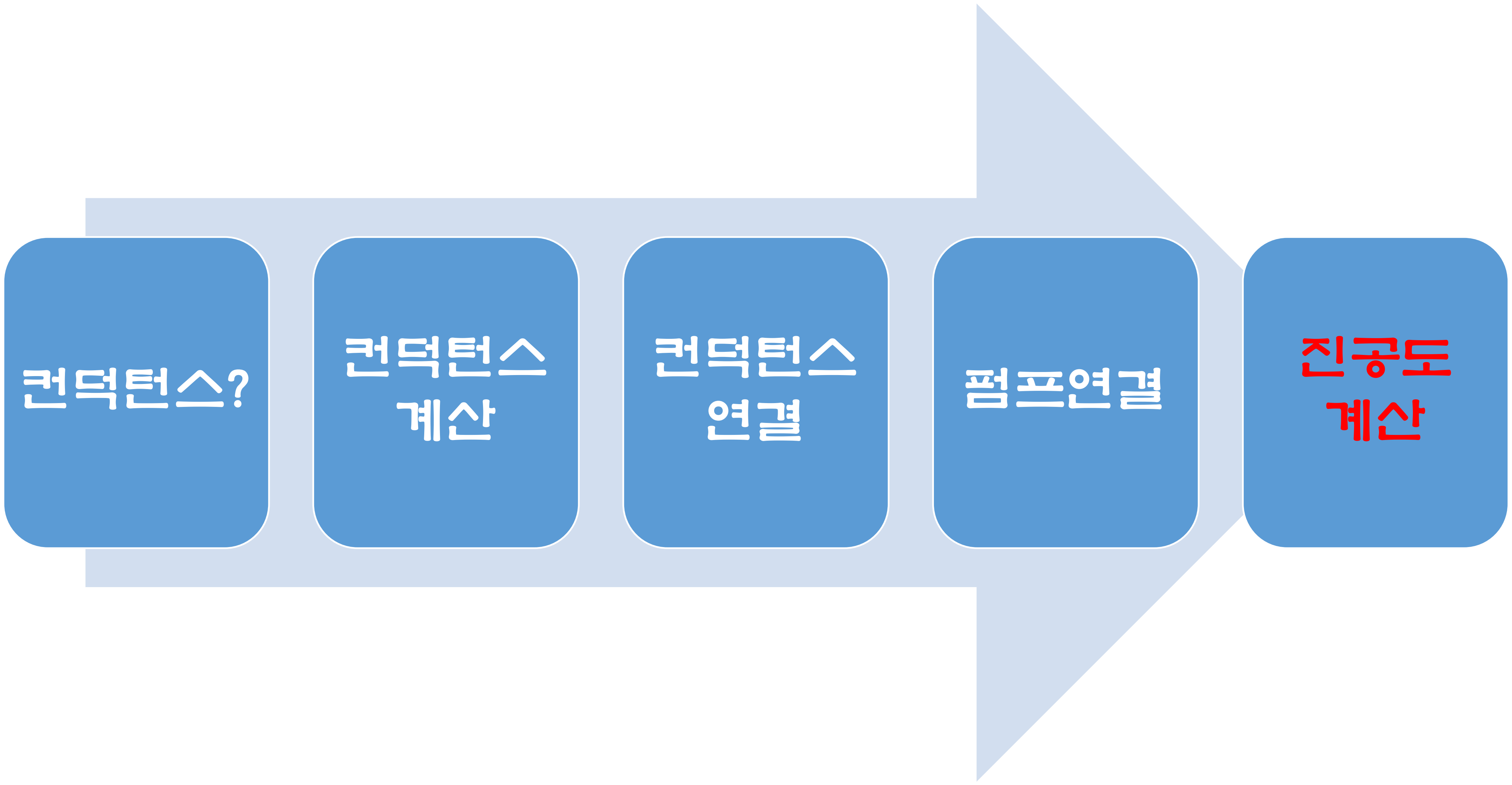


예제

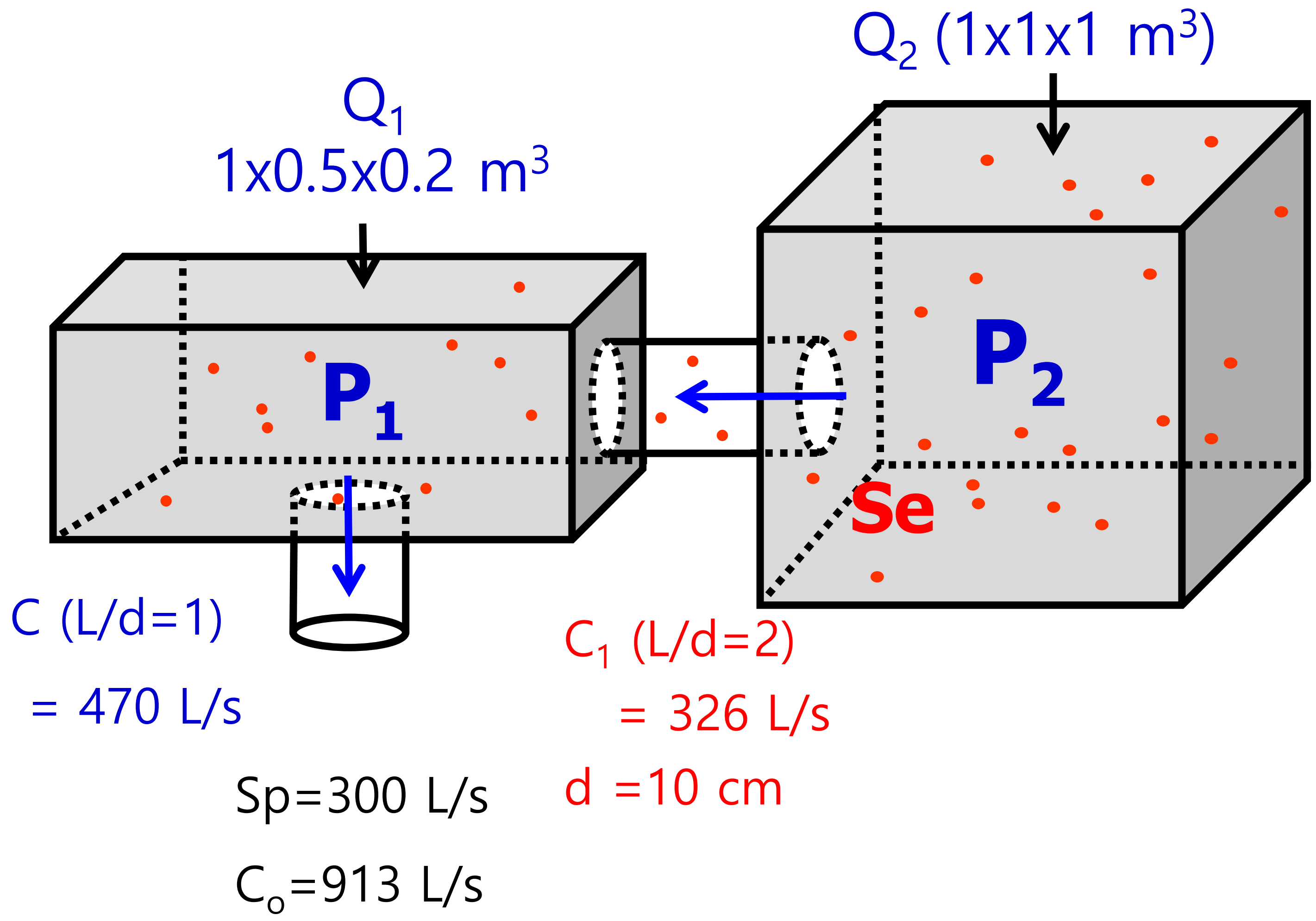


$$\frac{1}{\mathrm{Se}} = \frac{1}{c} + \left(\frac{1}{500} - \frac{1}{\mathrm{C_o}}\right) = \frac{1}{155} + \left(\frac{1}{500} - \frac{1}{913}\right)$$

$$\mathrm{S_e} = 136\ \mathrm{L/s}$$

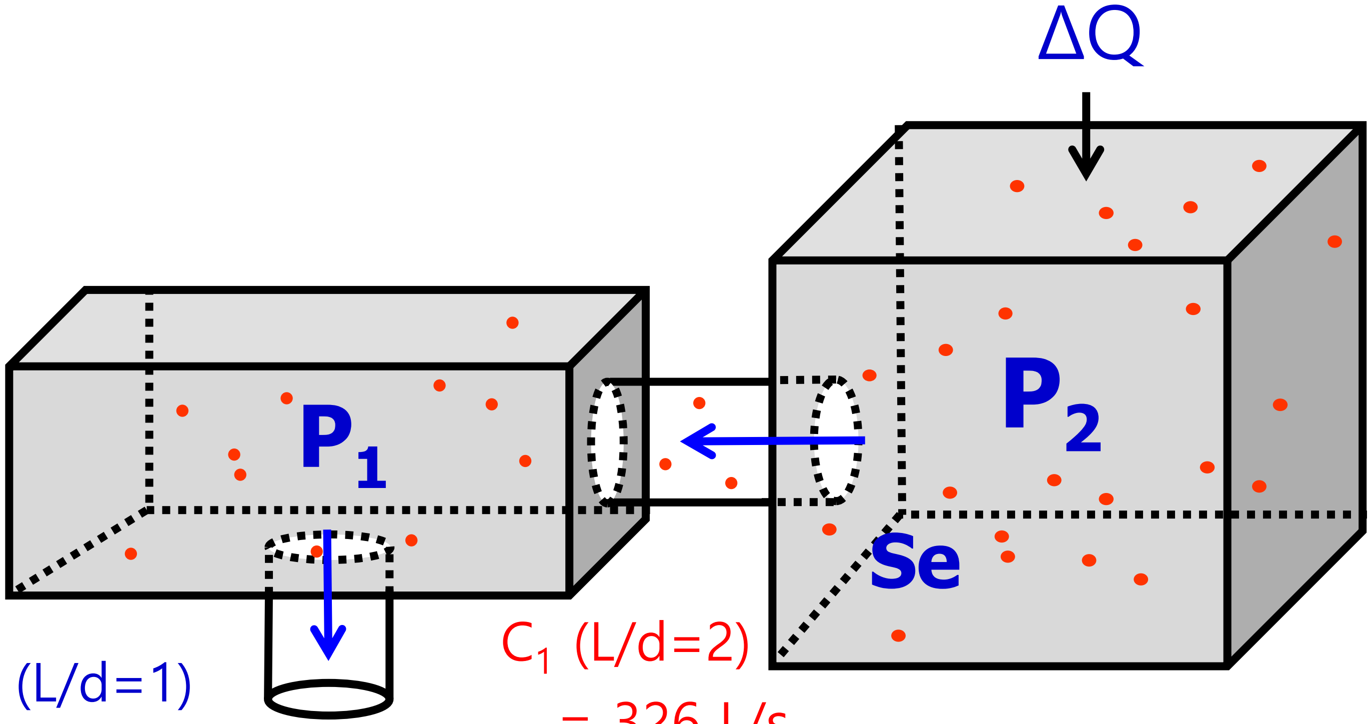

컨덕턴스?
컨덕턴스 계산
컨덕턴스 연결
펌프연결
진공도 계산

진공도 계산: 예제



Q2 (1x1x1 m3)
Q1
1x0.5x0.2 m3
P1
P2
Se
C (L/d=1)
= 470 L/s
C1 (L/d=2)
= 326 L/s
d =10 cm
Sp=300 L/s
Co=913 L/s

1. 기체 주입



C (L/d=1)

= 470 L/s

C_1 (L/d=2)

= 326 L/s

d =10 cm

Sp=300 L/s

C_o=913 L/s

Q=3.84x10^{-5} mbar·L/s, 23^{o}C

$$\frac{1}{S_e} = \left[\frac{1}{C_1}\right] + \left[\frac{1}{C} + \left(\frac{1}{S_p} - \frac{1}{C_o}\right)\right]$$

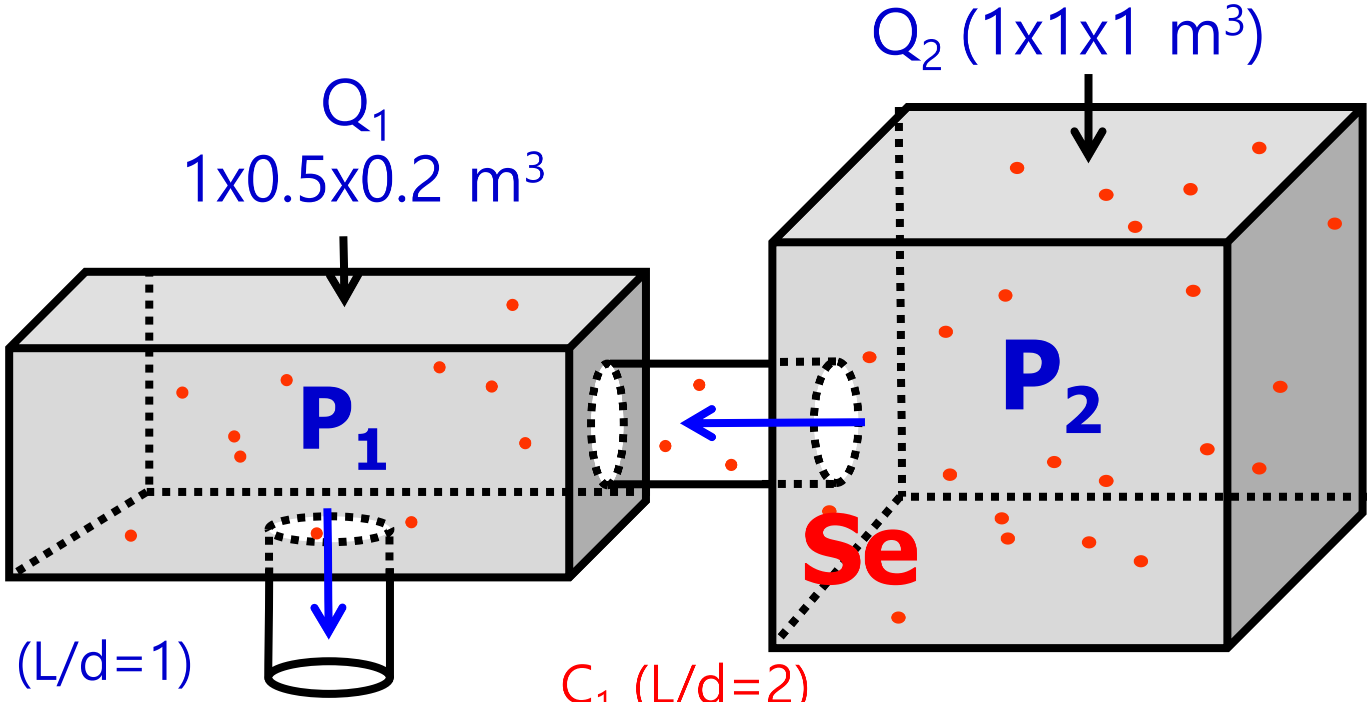
$$P_2 = \frac{Q}{S_e} = \frac{3.84\text{x}10-5}{134}$$

$$= 2.85\text{x}10^{-7}\ \text{mbar}$$

$$P_1 = \frac{Q}{S'} = \frac{3.84\text{x}10-5}{229}$$

$$= 1.68\text{x}10^{-7}\ \text{mbar}$$

2. 표면배기



C (L/d=1)

= 470 L/s

C_1 (L/d=2)

= 326 L/s

Sp=300 L/s

d =10 cm

C_o=913 L/s

$Q_{air}=q\cdot A=5\times10^{-10}\cdot A$ mbar·L/(s·cm^2), 23^{o}C

$$\frac{1}{S_e} = \left[\frac{1}{C_1}\right] + \left[\frac{1}{C} + \left(\frac{1}{S_p} - \frac{1}{C_o}\right)\right]$$

$$P_2 = \frac{Q_2}{S_e} = \frac{3.01\times10^{-5}}{134}$$

$$= 2.24\times10^{-7} \text{ mbar}$$

$$P_1 = \frac{Q_1+Q_2}{S'} = \frac{Q_1+Q_2}{229}$$

$$= 1.81\times10^{-7} \text{ mbar}$$

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